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ABSTRACT

This volume is based on reports from representatives of eight countries concerning current mathematics curricula for the primary grades with emphasis on inclusion of topics in "new mathematics." The eight countries are: Austria, Belgium, Switzerland, Federal Republic of Germany, France, United Kingdom, Ireland, and Finland. A chronological outline of reform in mathematics education in these countries is presented. The major portion of the volume is organized around five major topics: curricula, teaching methods, teacher characteristics, implementation of new curricula, and evaluation. Within each of these areas, several subtopics are examined, and the practices of the eight countries are compared. Two extensive reports are presented as appendices. The first concerns a case study of teaching methods in England; the second deals with teacher training in West Germany. (SD)

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COUNCIL OF EUROPE

Committee for General and Technical Education

THE TEACHING OF NEW MATHEMATICS IN THE PRIMARY SCHOOLS OF EIGHT COUNTRIES OF EUROPE

Consolidated report by S. ROLLER

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1 INTRODUCTION

Towards the end of November 1959, it may be said that an explosion took place at Royaumont. It was one that had been brewing for a long time, and its artificers were many.

First, there had been the mathematicians:

- Evariste Galois and the theory of groups
- Georg Cantor and the theory of sets
- David Hilbert and the formalisation of geometry
- Nikolaï Ivanovitch Lobatchevski and Bernard Riemann and the non-Euclidean geometries
- Lastly, the "all-rounder" Bourbaki and the unitary build-up of mathematics.

There had been a thorough overhaul of the experimental method, the inductive approach of Francis Bacon being replaced by a form of neo-Keplerism in which scientific research is based on explanatory models that are the more effective for being structured on mathematical principles.

There had been Jean Piaget's epistemology of mathematics, the ultimate fruit of lengthy work on genetic psychology which revealed the gradual construction of logico-mathematical thought from the child's earliest years.

There had also been the psychologists and psycho-pedagogues — Froebel, Binet, Decroly, Kerschensteiner, Dewey, Claparède — to demonstrate that in the final analysis a child "knows" only those things that he has himself experienced or "acted out" of his own free will.

Finally, to complete the lighting of the fuse, there had been the vast weight of a technology worked out by computer, stimulated by the demands of a world war and further whipped up by the needs of an expanding economy.

The Royaumont seminar touched off a spectacular renewal of mathematics teaching. It had been convened by the Organisation for European Economic Co-operation.

The meeting of experts held at Strasbourg on 18 and 19 December 1973 was called by the Committee for General and Technical Education of the Council of Europe. Moving from the economic to the political, and thence to educational affairs "in general", those attending felt the need to establish a "balance-sheet". This led to the theme of the reports drawn up a year ago, namely: "Comparison of the results at national level of the introduction of new mathematics into the primary-school curriculum". The reports, eight in number, were the work of the following authors:

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Hence, finally, the present report:

"The teaching of new mathematics in the primary schools of eight countries of Europe."

It is not now a question of commenting on *results* in the field of new mathematics teaching. Rather, what is presented is, a *situation report:* for, after more than 15 years of hard work it must be admitted that the time for assessments has not yet come. Research still forges ahead: the reports bear witness to that. They have not been prepared in accordance with any standard formula. This means a snag in the sense that they are somewhat disparate, and it has not been easy to process the data they contain. But their very disparity has its advantages too, in that each report has given, for the country concerned, a clear, authentic picture of mathematics teaching, even if at times the picture tends to be incomplete.

Curricula or programmes?

At the meeting in December 1973 there was discussion of the "curriculum". This term would seem to have been still considered as synonymous with "programme", but nevertheless with a nuance: the experts went beyond programmes — syllabuses — and spoke of methods, media, teacher-training, assessment. They do not appear, for all that, to have viewed their discussions in the theoretical framework of "curriculum development" or "Curriculum-Forschung". I have thus felt justified in this report in making considerable use of the word "programme".

My working method was as follows: after a first reading of the 8 reports, a series of headings was drawn up for my analysis, on the lines indicated in the heads and sub-heads of chapters 2 to 6. Since it was modified as the work proceeded, it is in fact co-extensive with and not anterior to my report. This was an empirical rather than a theoretical approach that was forced upon me by very reason of the wide diversity of the texts under study.

2 BACKGROUND TO REFORM

The most striking events contributing to the renovation of mathematics teaching have been pointed out by the rapporteurs because of the importance they attach to them. A near-exhaustive list of them follows.

1958 CH	Reform of maths teaching in secondary schools begun.
1959 OEEC	(23. 11-4. 12) Royaumont seminar on "New Mathematics" organized by the *Organisation for European Economic Co-operation* (OEEC) Extracts from the Contents table of the report: — Arguments for reform — Proposals for reform — Reforms finally proposed — Place of mathematics in education — Training of teachers — Programme, supervision and teaching aids — Syllabuses and examinations
1960 A	Some interested teachers in primary schools started to experiment with the tasks and exercises suggested by the "Theory of sets", examining largely the theses of Z.P. Dienes and the proposals of German working groups. The experiments in the USA, England, France and Belgium were followed closely.
1966 F	The "Lichnerowicz Commission", a ministerial commission on mathematics teaching, was set up. It prepared the "syllabuses and commentaries" to be officially communicated to schools on 2. 1. 70.
1967 SF	Proposals of the Nordic Committee for Mathematics Teaching, based on the work of the Royaumont seminar.
1968 D	The Standing Conference of Ministers of Education of the *Länder* adopted the new programmes proposed by the ad hoc commission set up in 1965.

	F	*Chambéry Charter* Published by APMEP (Association des professeurs de mathématique de l'enseignement public). Aims and methods of new mathematics teaching.
	IRL	Launching of the new mathematics syllabus to be adopted in 1971.
1969	F	Creation of the IREM (Instituts de recherche sur l'enseignement mathématique). There were 3 at that time; by 1973 there were 21.
1970	A	Experiments with new teaching methods (Prof. Ingo Rath) in 7 grades in *Land* Salzburg.
1970	F	The reform of mathematics teaching — course content and teaching method — in the primary school (6-11 years) was outlined in the "*syllabuses and commentaries*".
	SF	A new curriculum for the comprehensive school was published. It has been implemented gradually in the primary schools as they transfer to the comprehensive system.
1971	B	Ministerial circular to launch the experiment on the teaching of the new mathematics syllabus in primary schools.
	IRL	Course in new mathematics prescribed as an integral part of a complete new child-centred primary-school curriculum.
1972	A	Publication of a new mathematics teaching programme. Application of new teaching methods in schools covering 30% of 1st- and 2nd-grade pupils.
	CH	Adoption, by the French-speaking cantons, of the "Syllabus for primary schools in French-speaking Switzerland", comprising a completely remodelled mathematics syllabus.
	D	Application of the new 1968 syllabuses.
1973	A	Application of the new mathematics syllabus in the first grade.

CH Introduction of the new mathematics syllabus in all first-year grades (ages 6-7) in French-speaking Switzerland.

Remarks

The history of the reform is marked by 4 phases:

1. Royaumont seminar, 1959-61. Growing awareness of the economic, technical, scientific and cultural need to rethink the mathematics syllabuses.
2. Preparatory phase, 1964-67:
   Creation of working groups:
   GB: Project "Mathematics 5-13"
   D: Schoene Committee
   F: Lichnerowicz Commission
   SF: Nordic Commission for the Teaching of Mathematics
3. Promulgation of new programmes and experiments, 1968-71
4. Gradual generalisation of the new teaching, 1972-73 . . .

3 CURRICULA

3.1 Bodies responsible for reform

A — *Federal Ministry for Education and Art*
Executive body: "Centre for School Experiments and Development"

B — *Ministry of Education and French-language Culture*
Responsible body for preparing syllabuses, among other activities: "National Education Commission"

CH — *Departments of Cantonal Public Instruction*
Almost all have entrusted the organization of reforms to persons appointed on an ad hoc basis or to research bodies

F — *Ministry of Education*
Executive bodies: the "Institut national de recherche et de documentation pédagogique" (INRDP) and the "Instituts de recherche sur l'enseignement mathématique" (IREM)

GB — *Local education authorities;* school governors and managers; head teachers (who make the actual decisions)

IRL — *Department (Ministry) of Education*

SF — No special mention. The report refers to the part played in the reform work by the "Institute for Educational Research"

Remarks

The information given above reflects fairly closely the way in which school questions are managed in the various countries. The system may be either centralized or decentralized. It is clear too that the reform is not part of a general framework embracing the entire school world, based on long-term planning. Reform has been, and in some places remains, of a regional and experimental character.

3.2 General characteristics of the new programmes

3.2.1 *Place of the new mathematics syllabuses in the general school curricula*

Mathematics teaching reform is part of a general renovation process that affects all school systems.

In Belgium, the "Provisional mathematics syllabus" is a component part of "reformed primary education".

In French-speaking Switzerland, the mathematics syllabus for grades 1 to 4 of the primary school appears in the "Syllabus for primary education in French-speaking Switzerland" adopted in December 1972.

In France, there is insistence on the fact that the reform of mathematics teaching goes hand in hand with the reform of primary education *as a whole*.

The same is true of Ireland, where it is pointed out that the new syllabus is an integral part of a complete new child-centred curriculum. It would even seem that, in that country, it was the very desire for a "child-centred" education which led to including new mathematics in the subjects taught.

3.2.2 *General aims of the new syllabuses*

— *Mathematical aspect.* The Belgian report urges that pupils be initiated into the present conception of mathematics which, while accepting the heritage of the past, cannot ignore the most recent discoveries, these being, moreover, particularly fruitful in their application to the modern world. Hence the need to "mathematize" young minds. This will be done, for a start, by mathematizing "certain knowledge common to and spontaneous in children" — knowledge containing in embryo some of the fundamental mathematical concepts. "Education" says the report "must build the house of mathematics for children of 6 to 12".

— *Logico-mathematical aspect*

Belgium:

"One of the aims of all mathematics teaching is to teach the child to reason logically. This is a faculty that is to a great extent transferable to other disciplines and to many activities of everyday life, provided that the reasoning processes have been sufficiently clearly presented."

Switzerland:

"Mathematics teaching at primary school should:

— inculcate sound mental processes; that is, develop logical reasoning, an ability to locate, classify and arrange and an ability to understand and depict a situation;

— give a good intuitive grasp of the fundamental concepts: sets, relations, operations, structures;

— provide an intellectual tool capable of being used in the most diverse situations of everyday life;

— develop adaptability and inventiveness."

— *Psycho-pedagogical aspect*

The Irish report, stressing the wish that the new teaching should be "child-centred", mentions the acceptance of Piaget's work on genetic psychology and urges the need to kindle a lively interest in mathematics in the child.

3.2.3 *Internal structure of the new syllabuses*

— *"Indicative" programmes*

The desire has been expressed — in Germany, France, the United Kingdom and Ireland, in particular — that the syllabus should allow reasonable flexibility. The "curricula" in the Fed. Rep. of Germany have been the subject of a *recommendation* by the Standing Conference of Ministers of Education of the *Länder:* this has been used as a basis for each *Land* to work out its own syllabus.

The French syllabus — which is deliberately *short* — is designed to leave teachers the greatest possible liberty; such liberty is, indeed, inherent in the reform as a whole.

In Ireland, "while the syllabus in each section might be accepted as a *general indication* of what ought to be undertaken at each stage, it is primarily intended as a guide to the sequence of concept development . . . The syllabus should be adapted to suit the needs of schools, classes and individuals".

"Mathematics" says the British report "is part of a child's heritage, but how difficult it is to select from this wide field the small amount that can be offered in the primary school. Perhaps it can only be hoped that they will enter their secondary schools with a feeling for the integrity of the subject, competence in the skills learnt, minds open to new ideas, enthusiasm for the subject and a desire for a better acquaintance with it."

— *Syllabuses based on "objectives"*

Prof. André Delessert (Lausanne University) addressed Swiss teachers in 1970 on the introduction of "objectives" into new mathematics teaching, as follows:

"Traditional teaching" he said "was defined in terms of 'syllabuses'. Today's teaching is determined by *objectives* for thought and action. The primary aim is no longer to study triangles or vulgar fractions, or vectorial space: it is to learn how to inventory, classify, arrange, choose, decide, criticise and correct — and also to acquire the desire for understanding."

This notion has since been generally accepted, as witness the syllabus for *Land* Hessen (D), which appends to the syllabus drafted by the *Kultusministerkonferenz* a detailed list of *objectives* formulated in behavioural terms. Example taken from the syllabus for the *first year*, chap. 1 (Sets), sec. 1.1 (Things and their properties):

1. to get familiar with structured material by playing;
2. to sort blocks (e. g. logical blocks of Dienes) according to their properties;
3. to be able to name properties of blocks;
4. to find things with prescribed properties;
5. to find a law in a series of things;
6. to form a series of things according to a prescribed law.

Note: Objectives 5 and 6 enable groups of children to be formed in accordance with their aptitudes.

— *Flexible syllabuses*

The necessarily schematic presentation of content must be accompanied by certain checks and balances so as to preserve the liberty inherent in the new conception of mathematics teaching and the considerable flexibility it must enjoy.

The Belgian report, after listing the 6 basic themes of the syllabus, adds:

"These themes are not kept in watertight compartments; they are designed to help each other, for the greater benefit of the unitary conception of mathematics and the daily 'menus' served up."

For the French-speaking Swiss, the four "avenues" that constitute the framework of their syllabus (Sets and relations, numeration, operations on cardinal numbers, discovery of space) are not tackled in succession but in parallel. Moreover, the programme is *cyclical*:

> "A concept studied at a given time reappears periodically to be re-worked, studied in depth, expanded. This enables certain children who develop at a different rate from the majority to catch up with the others, without the latter feeling themselves 'held back'."

The Austrian report, too, speaks of a similar "cyclical" teaching, which it calls the "curriculum spiral" (term used by J. S. Bruner).

3.2.4 *How the new syllabuses are prepared*

Almost every new syllabus has been the subject of experiment prior to its promulgation. Although some reports make no mention of this, the reason is doubtless that there can no longer be any question of proceeding in any other way.

It is also generally agreed that the new programmes are merely "provisional" (Belgium), "experimental" (Switzerland) or "transitional" (Ireland, France). What we are witnessing is thus a process of continuous innovation.

3.3 **Content**

3.3.1 *Synthetic presentation*

Content varies widely in the different reports. Some go into great detail (Belgium, e. g., appends its 1971 "Provisional mathematics syllabus"); others give no indication (Finland). The following list is thus far from exhaustive. It gives the most important of the data furnished by the rapporteurs, but can be no more than "indicative" — a signpost to "trends". Where no mention appears (absence of the + sign), it should not be interpreted as a defect inherent in the syllabus. It simply means that the rapporteur has not felt the need for any special reference to the point.

SETS AND RELATIONS

	A	B	CH	D	F	GB	IRL
Sets	+	+	+	+	+ 1/	+	+
Relations	+	+	+				
Proportionality					+		
Structures (of groups)		+					

1/ F – Classifying and sorting activities

NUMERATION	A	B	CH	D	F	GB	IRL
Cardinals	+	+	+	+	+/3	+	+
Positional notation	+		+	+	+	+	+
Bases	—1/						+
Decimal system		+2/	+			+	+
Binary system		+					
Prime numbers							+

1/ A It is specified that grouping operations, to the base 10 or any other base, are omitted.

2/ B The Belgian report, objecting to a "recommendation" of 19. 12. 73 by the Committee for General and Technical Education (introduction in the 1st primary grade – age 6 – of numeration systems to bases other than 10), considers that such other bases should not be introduced at this early stage. *Decimal* and *binary* notation should suffice.

3/ F Number as a property of a class of equipotent sets.

RATIONAL NUMBERS	A	B	CH	D	F	GB	IRL
Rational numbers		+		+1/			
Vulgar fractions	+	+				+3/	+
Numbers with decimal point					+2/		

1/ D Grades 5 & 6

2/ F Revised presentation of fractions

3/ GB Equivalence categories

MEASURES	A	B	CH	D	F	GB	IRL
Measures	+			+	+		+2/
Metric system	+	+				+1/	
Time measurements	+						

1/ GB Initiation into the metric system, a new departure in the UK. The fact that the metric system co-exists with the older units of measure will be a help to the children in mastering the idea of conservation.

2/ IRL Activities connected with the children's environment.

OPERATIONS	A	B	CH	D	F	GB	IRL
Operations on cardinals	+1/		+	+	+		+4/
Addition	+	+			+	+	+
Subtraction	+	+				+	+
Multiplication	+	+				+	+
Division	+	+		+2/		+	+
Addition & multiplication tables	+					+3/	+
Mental arithmetic	+			+			

1/ A The teaching of new mathematics has not displaced certain notions proper to traditional instruction — e. g. a sound training in arithmetic.

2/ D Divisibility, set of divisors. Grades 5 & 6.

3/ GB Number bonds up to 9 x 9.

4/ IRL "Arithmetic" has preserved all its importance. Arithmetical skill has always been keenly striven for in Ireland, which has been proud of the standard reached by pupils.

DISCOVERY OF SPACE	A	B	CH	D	F	GB	IRL	SF
Exploration of space	+	+	+		+	+	+2/	
Topology	+							
Geometry	+	+		+				+
Graphs, diagrams						+1/		+

1/ GB From concrete objects to diagrams. Problem-solving with the aid of line-graphs.

2/ IRL First experiments with three-dimensional objects.

PROBLEMS	A	B	CH	D	F	GB	IRL
Applied maths		+					
Problems					+1/		
Social arithmetic							+2/

1/ F Recurrence of similar problems and methods of solving them.

2/ IRL Calendar, timetables, simple interest, profit & loss.

ALGEBRA	A	B	CH	D	F	GB	IRL
Algebra						+1/	+2/

1/ GB This form of algebra, which appears on the primary-school syllabus, mainly consists of "generalisations" reached by the children on the basis of concrete situations.

2/ IRL At "senior level", algebra up to simple equations and inequations.

3.3.2 *Commentary on syllabus content*

Despite all the — unintentional — gaps in the above analytical tables, the trends of which we spoke may be clearly discerned:

— The new ideas exist side by side with the old. This, indeed, conforms to the wish expressed by the Committee for General & Technical Education at their meeting on 19. 12. 73, when the experts concluded that "pupils must have an easy and accurate command of mental and written arithmetic in the decimal system". The Belgian report states that "arithmetic" must not be regarded as an end in itself, but must always be seen as an instrument whose correct use must be understood by the pupils as an adequate means of solving problems put to them or imagined by themselves.

The move from the old to the new syllabus is not without its stumbling-blocks.

The Austrian report, after recalling the content of traditional *arithmetic* teaching, notes that the objectives set by the former syllabus have, until now, been *attained*. "Many teachers and parents therefore have the impression that something tried and tested is being given up for something uncertain."

— Ideas of "sets" are suggested as a means of exercising logical thought: membership of a group, inclusion, joining-up, intersection, etc. There is no claim that children are being initiated into the "theory of sets" — a claim that is in any case condemned by mathematicians.

— Number is more and more considered as a property of a set, and there is a general trend towards calling it a "cardinal".

— Numeration is based on the general notion of the positional notation. The use of bases other than 10 is in order, since it has inter alia the advantage of familiarising pupils with the notion of "exchange", a valuable element for the future when they are learning the techniques of the four operations. It is felt, however, that the base 10 should be given a privileged place, and perhaps also the base 2 (computers).

— Fractions are losing ground. They are approached as "rational numbers". Numbers with points — to other bases — are a new way of viewing fractions.

— Measures are of a functional character: they bear reference to the unit selected. The metric system is gaining in importance in the British Isles.

— Teaching the operations makes it possible to deduce general properties, such as associativity, commutativity, distributivity. Thus the pupil should always be capable of reconstructing the machinery of multiplication, for example.

— Addition and multiplication tables are still considered as basic knowledge. But learning them today is no longer a dull parrot-fashion process; it now rests on reasoned construction.

— Geometry, hitherto reserved for the higher grades of the primary school, is now already encountered in grade 1, under the title of "discovery" or "exploration" of space. It is accompanied or preceded by topology. Here teachers are in agreement with Jean Piaget, who has demonstrated that children already think along topological lines before entertaining notions of classical, not to say Euclidean, geometry.

— Pictorial representation, 3- or 2-dimensional, of daily-life situations plays a large part in the English syllabus. It is held to be a way of linking school with life.

— Those hardy annuals, "problems", are still with us, but have been given a new look. They must emerge from the child's surroundings and be posed by the pupils themselves; several solutions, or even none, should be possible . . . They provide opportunities for group work and discussions.

4 METHODOLOGY OF TEACHING

The experts who met in Strasbourg on 18 & 19 December 1973 decided to ask the United Kingdom for a "case study" of the methodology of the new mathematics teaching. The study would contain information from the projects launched by the Nuffield Foundation and the Schools Council, and should cover, inter alia, the following aspects of methodology:

- — formalising of experiences of the young child;
- — introduction of environmental material;
- — formation of ability groups of 6 or 8 children for the lesser or highly gifted;
- — if possible, statistics of results obtained from different methods of approaching a given problem.

4.1 General principles

It would seem from a reading of the reports that *learning mathematics* is not so much a problem of *instructing* the individual (supplying intellectual baggage) as a problem of *education*. Above all, the rising generation must be imbued with a *mathematical spirit* comprising a taste for operating, in all things, according to the rules of strictly logical reasoning, and must be given the means for doing so. Such, in the view of the experts meeting at Strasbourg in December 1973, is the essence of the reform movement.

In addition, it is hoped that the new teaching will be *coherent* in the sense of the wish formulated by Madame Nicole Picard in an article published by the Swiss review "Math-École" (Jan. 1967): "If mathematics teaching" she wrote "is to be coherent — i.e. if it is not to need a constant 're-run' or to suffer from slackening impetus — it must start in nursery school and continue *in the same spirit* (though not, of course, the same manner) right up to university."

From the psychological aspect, there is insistence on the children's *motivation* (functional standpoint), and especially on the need to bear in mind the laws of psychogenetic development put forward by Jean Piaget, whose "constructivism" is highly regarded: it is, indeed, at the root of all recommendations that give pride of place to the activity of the child. This is a logical sequence to the "active school" and its insistence on action by the child (Alfred Binet said, around 1910: "The child knows only those things which it has done on its own initiative").

"The learning of mathematics" continues N. Picard in the same article "will henceforth be based on the child's own creative activity. The children must construct their own mathematics by working out its concepts for themselves: they must be given the opportunity of *abstracting* (in the etymological sense of the term) those concepts from their own experience. In this way we may also, perhaps, enable them to construct their own personalities."

Thus it is less a matter of initiating pupils into an already cut-and-dried mathematics than of helping them participate in "mathematics in the making". In fairness to them, however, if "the spirit of mathematics" is to be safeguarded, the teacher must keep his presence of mind and, from the outset, not delude himself into thinking that the child, as it were by magic and by its own manipulations alone, will develop a mathematical outlook. Speaking of the material used in the classroom, the Belgian report, on this point, is categoric: "The material may be the departure point or the medium, but never the point of arrival, otherwise we shall end up with a 'manualism' that is both sterile and disheartening."

We may usefully recall here the distinction made by Jean Piaget between two kinds of abstraction, the empirical and the reflective. Knowledge, in the view of the leader of the Geneva school, is in the first instance little more than a reading of the ambient environment. Let there be no mistake: that reading does not simply reflect a passive approach by the individual in the sense of being "impressed" by reality like a photographic plate. At that level, abstraction already exists — empirical abstraction — since the individual knows little of the environment other than its real or potential effects upon himself. The initial knowledge thus gained ensures that there will be instant, practical apprehension of the situation. It will not go very far, because it cannot be easily transposed to other situations. As knowledge it is, in short, costly. Hence the individual, during his genetic evolution, perfects a fresh form of cognitive behaviour — reflective abstraction — which, using the empirical approach as a stepping-stone, reconstructs it and raises it to a level of generality which allows him to stand aloof from the immediate environment to which the empirical abstraction was attached, in order to adapt himself to a richer and more varied reality (1).

At the level that concerns us here — the level of concrete operations which takes over from the sensory-motor level — we are dealing with empirical abstraction. For all its empiricism, this is nevertheless a true abstraction, with its characteristic features and demands that presuppose the functioning of thought, thought which already guides action. Objects or "situations" thus have a value only in so far as they throw down the gauntlet to the mind, challenging it to invent the necessary steps to be taken, whose validity it will then check by testing, through conscious manipulation, the means placed at its disposal.

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(1) Notes written on reading Piaget's recent work "Adaptation vitale et psychologie de l'intelligence". Hermann, 1974.

4.2 Methodology of the reformed teaching

4.2.1 *Prerequisites*

Children, says the Austrian report, cannot be expected to learn mathematics until they have been prepared for it.

From the earliest age (about 5), add the British, "we must ensure that we create an environment which encourages [children] to flourish and to develop both their enjoyment of and their skill in the subject".

4.2.2 *Games and functional approaches*

To motivate the children, an important part must be allotted to games, since these are a vital need, at least for the youngest. Later on, the various activities lose their play element to some extent and take on a teleonomic character which brings them closer to actual work: the situations facing the pupils, and the activities that will derive therefrom, must be made meaningful. The child must understand the reason for what he is doing.

4.2.3 *Situations*

"Situations" are part of the jargon of the new mathematics teaching. They are universally recommended, and, like "manipulation", an aura almost of reverence surrounds them.

It is important, says the British report, to give children the opportunity of *experimenting* in an environment rich in *situations* which will lead them to investigate. These situations represent so many stimuli which encourage multi-dimensional responses, imagination and creativity. They must, however, be *meaningful;* they must correspond to a need felt by the child and awaken a powerful interest. The best situations will be those that occur in the practical daily life of the children.

"It would be a pity" says the Belgian report "if advantage were not taken of the mathematical facts already to be found in the common knowledge of the children — recollections of experiences in or out of the classroom, which will be called to mind, polished up and enlarged upon. The resulting awareness will enable the child to acquire a relative mastery of certain ideas — an acquisition reflected in the ability to utilise these ideas in new situations."

However, daily life does not necessarily offer all the desired opportunities for stirring up the intellectual processes to which the children must, after all, be subjected. Hence this recommendation of the Belgian report:

"There are some notions essential to the building of the house of mathematics which are not to be found in the children's common knowledge. The teacher then has to imagine simple situations or mathematical games which highlight these notions and which serve as a springboard for constructing them."

4.2.4 *Manipulation and creativity*

The "Teacher's Handbook" on the new mathematics teaching in 1st-year grades in French-speaking Switzerland is particularly explicit on the subject of "manipulations":

"In mathematics lessons, the mistress will take care to accord an important place to intuition, investigation and manipulation; against that background, use of work-books is of secondary interest. Psychologists have shown that intelligence derives from action; it is not just a matter of repetition, as was long thought, but is essentially active. Thus it is by personal construction, and not by imitation and repetition, that the pupil can acquire mathematical notions. To help this process, the child must therefore often be allowed to handle objects and establish relationships between them: mathematical laws are thus gradually discovered and then consolidated, by means of diverse experiments. Since certain elementary logico-mathematical ideas are drawn, not from the objects themselves, but from actions which the child exerts upon them in co-ordinated fashion, it follows that a teaching aid is of value only because of the reflections it awakens."

Thus the child is induced to invent his own mathematics, to shape it for himself.

As the French report observes: "On the basis of situations usually prepared by the teacher, and with the help of manipulation and reflection, the child should himself arrive at the concepts which it is desired he should learn."

Nevertheless — and we agree with some of the rapporteurs that this point needs to be emphasized — the activity stimulated by situations should not be merely a manipulatory one, calling for immediate practical action. "Thus" says the Belgian report "in his introduction to mathematics, each child will be helped to go beyond *situations calling for immediate action* in order to raise himself, as far as possible, to a higher level: the use, for his own ends, of the knowledge and know-how acquired."

In manipulations of objects, primary importance attaches to acts — acts fully comprehended and co-ordinated. The actual result of these acts — materials arranged in order, drawings produced — is of secondary value only, serving merely as a check on the correctness of the act. Unless care is taken, the materials to be manipulated acquire a kind of separate existence and the images to which the activity has given rise are likely to be "read" independently of the "mental" activity which was to have directed their production. The threat is then the emergence of a new verbalism, the verbalism of images, even if these consist of large three-dimensional objects.

4.2.5 *Abstraction*

The process of abstraction is one of the characteristic workings of the human mind, its end result being to furnish the individual with mental tools capable of responding to a very large, not to say infinite, number of situations. "Through all the changing scenes of life", with its bewildering extremes, abstraction is a means of selecting the instrumental invariables needed for coping with any situation. Hence, if it is to be successful, there must be established between the subject and his view of reality a dialectic relationship, in the course of which the individual fashions certain models — in themselves embryo abstractions — and tests their reliability by fresh reference to that reality. This is well expressed by the Belgian report:

"We believe that this climb (towards the abstract) does not take place in distinct stages interrupted by long intervals, but rather results from short and frequent interchanges between situations 'lived' and the general idea that endows them with unity: between the contingent and the corresponding structure."

The interchange — a mind-training phenomenon — between an inquiring thought and a reality that needs explanation or, at least, orderly arrangement, may include intermediate stages. Pictorial representation may be a substitute: "Certain mathematical concepts are discovered and exploited by interpreting simple pictorial representations, judiciously selected by the teacher." (Belgian report)

Thus on the one hand reality itself, and on the other hand pictorial representations, which in relation to reality play the rôle of signs, are so many "perceptive situations" in the meaning accorded to them by Z. P. Dienes. But, once again — and the various reports would seem to bear out this view — perceptive situations are, in themselves, powerless. Their sole raison d'être lies in stimulating an activity which, though in the young it may be heavily saturated with sensory-motor elements, assumes its full value as a mathematising factor only after it has become part and parcel of the mind and more and more takes the initiative in the requisite operations.

4.2.6 *Verbal expression*

The rapporteurs have not overlooked this aspect of mathematical training. The British report stresses the importance of *mathematical language* and of progress from things to symbols first by mastering that language orally and then by going on to pictorial recording and subsequently written sentences. The Swiss report speaks of "operations on cardinal numbers" on the following pattern:

"— manipulation with various materials,
— verbal expression,
— symbolisation without the help of any material,
— written expression.

When it comes to practical applications, inventiveness should always precede the setting of exercises."

The time seems past when it was believed that language was among the constituent elements — a causal element — of reasoning. The work of Madame Hermine Sinclair (School of Psychology and Educational Sciences, University of Geneva), though never mentioned by the rapporteurs, is implicit in their reports. "Rational" language, the language used to express mathematics, emerges gradually as the processes of logico-mathematical thought take shape. It contributes towards strengthening those processes, but it is not their cause.

4.2.7 *Individualisation*

The principle of individualising school work is universally accepted. The reports say little about it, although this principle is especially valid for mathematics teaching.

As we said earlier, *Land* Hessen in the Fed. Rep. of Germany has expressed the elements of its syllabus in terms of "behavioural objectives". Some of these objectives (e.g. to find a property common to a set of objects; to form a series of things in accordance with a given property) are used as *aptitude detectors* (a kind of tests) to allow the grouping of pupils by their abilities.

In the United Kingdom, "open-plan" schools group the pupils together by "bases" in accordance with their abilities, children of two or three age-groups being thus able to take part in the same activities. Moreover, the pupils are sometimes asked to plan their own work for the day, or even for a whole week.

Finland mentions the setting-up of small groups in order to give "supporting teaching" to the weaker pupils. The improvements made have, however, not gone much further than the automatisation of simple processes. It is therefore concluded that the attempt should not be made to teach everything in the new text-books: one should concentrate on the essentials. It will then be seen how each pupil may be offered what is particularly suited to his needs.

4.2.8 *The teacher's share*

The very large rôle now allotted to free investigative activities and creativity means, as the Swiss report says, a less rigid relationship between teacher and taught. The former abandons his magisterial position so that the latter may take the initiative, alone or in a group.

The British report points out that, if the teacher has had to come down from his desk to mingle with his pupils, the reason is that his *presence* remains especially important. "There must be teacher intervention and involvement, or intuitive ideas will not be made conscious."

The same report also discusses the respective value of "all-rounders" and "specialists" among the teaching staff. The former are especially suitable for younger children; they can adapt themselves to the needs of each pupil and closely follow his progress — e.g. the development of his inquisitiveness. The latter are particularly useful to more gifted children, who find in the specialist teacher much to stimulate their interests.

4.2.9 *Teaching media*

Text-books. As a means of transmitting, both to teachers and taught, the ideas officially written into the syllabus, text-books would appear to have retained their validity.

In the Fed. Rep. of Germany two types of work are currently found: old text-books "adapted" to the new requirements (the adaptations often leave something to be desired), and new works which, however, suffer from an inadequate experimental basis.

Austria has text-books and materials prepared by Prof. Ingo Rath and tried out experimentally in 1971-72 in three of the Federal States.

Belgium and Finland have been equipped with sets of new text-books.

In French-speaking Switzerland a series of *work-cards (fiches de travail)* has been prepared for pupils in grade 1 (age 6) by teams of teachers. The cards come into use only after manipulative activities which usually take place in groups under teacher supervision. Since the pupils draw and write on the cards, these have to be renewed each year. A teacher's handbook — "Notes on method" — goes with the cards. The work done for grade 1 (1st edition, June 1972) is being continued for the other grades (2nd year, June 1973, etc.).

In Ireland, four publishers have each produced sets of mathematical text-books based directly on the syllabus. Some are accompanied by pupils' work-books. The principal teacher of each school selects the set he considers most suitable to his particular school.

France, finally, points to the "surfeit" of manuals that have been published, each one trying to be more "modern" than the last. This, the report says, has had the effect of kindling the conflict between the supporters and detractors of "modern mathematics".

Materials

The educational value of "materials" to stimulate the pupils' activity is so generally recognized that the reports say little about it.

The British stress that, wherever possible, the starting-point must be the child's own environment. Occasions for exercising mathematical thought (sorting, classifying, arranging) will not be lacking in that environment.

The Belgians advocate simple, multi-purpose media for use by the teacher, such as diagrams, graphs, pictures and structured material. These should, however, be used with caution, in the sense of being guided by the child's needs. Materials may be departure or intermediate points, but never points of arrival. The report mentions the logical blocks of Z. P. Dienes, the coloured rods of G. Cuisenaire and G. Papy's Minicomputer.

Ireland also refers to the Cuisenaire material and "unifix" cubes, noting, however, that "few if any teachers use them exactly as their makers envisage".

The fact that the reports say very little about "materials" must also, we feel, be interpreted as a sign of wisdom. It is realised that no material is a panacea and that what is essential is the pupil's own activity, whose educative value will be all the greater to the extent that it starts from situations forming part of the children's lives and consistent with their inherent needs.

None of the reports mention the use of audio-visual media in the classroom. As will be seen later, television has been utilised for the re-training of teachers.

5 THE TEACHING BODY

Teacher training will also be the subject of a *case study* entrusted to Mr. Böddeker of the Fed. Rep. of Germany. It will cover such aspects as:

— modifications to initial training with a view to later re-training;
— content of initial or re-training of teachers for new mathematics;
— links between practical and theoretical training.

5.1 Profile of the new mathematics teacher

The "Ministerial Commission on Mathematics Teaching" (Lichnerowicz Commission) — F — writes in its final report: "The Commission would willingly go so far as to say that the teachers who dispense instruction in mathematics have a place among the most important men in our society, those who are closely shaping the future."

The harvest truly is plenteous, but the labourers are few. The French report indicates that only 14 % of primary-school teachers at present consider themselves sufficiently well equipped to tackle the new syllabus. Broadly speaking, French teachers desire more thorough training, more detailed syllabuses and instructions, advice on how the new syllabus is to be implemented, and one or more good manuals.

The German report also states that teacher-training is *the most important* aspect of the reform. The teachers, it adds, though often university-educated, have but rarely had occasion to acquire a thorough knowledge of mathematics.

The British report says much the same thing: "Many (of the teachers) will not have taken the study of the subject to the stage where they can deal adequately with the most able of our 11-year-olds." The fear experienced by some teachers that they will be unable to meet the new requirements is one of the main "deterrent" factors.

From all this it becomes clear that the new mathematics master must possess first of all a mathematical culture that will allow him to acquire that "mathematical mind" without which nothing good can be achieved.

Moreover, teaching which is voluntarily and, should we not add, compulsorily, "child-centred", presupposes that the same teacher will have had a sound training in psychology. The motto of Edouard Claparède's Institut Rousseau (Geneva 1912), "Discat a puero magister", could have been that of Jean Piaget. His studies in genetic psychology leading up to epistemology, especially that of mathematics, are the outcome of a lengthy "listening-in" to the child. It seems unthinkable that today's teachers cannot have been imbued with the constructivism of the leader of the Geneva school.

Only thus can the teachers, duly and suitably equipped with the requisite scientific baggage, make a positive contribution to the climate of liberty of which the French report speaks; for liberty in that sense is co-extensive with the entire renovation of mathematics teaching.

5.2 **Initial training**

Of this the reports say little, the main effort being for the present centred on re-training of teachers.

The French report is the most explicit on this point: Theoretical and practical training is given in the *écoles normales* for two years after the *baccalauréat*. The aim now is *self-training* based on the observation and analysis of teaching sequences and subsequent reflection.

5.3 **Re-training (In-service training)**

5.3.1 *Optional or compulsory?*

More often than not, in-service training is optional. It takes place partly during working hours and partly at other times (GB).

The Austrian report notes that, where the initiative was taken by voluntary working groups, the teaching of new mathematics achieved excellent results: children and parents were enthusiastic. Such is the power of motivation!

In view of the volunteer spirit prevailing, re-training in Austria is optional. This situation is not without its drawbacks since some teachers, who are uninformed, continue to throw discredit on the current reforms.

In French-speaking Switzerland, re-training started by being optional and, as such, was highly successful. The new maths teaching having been introduced in 1st-year classes in September 1973 (2nd-year classes in 1974, etc.), it became necessary to make in-service training of teachers obligatory.

5.3.2 *Duration*

The report by the meeting of experts (Strasbourg, 18 & 19 December 1973) points out that some teachers had had only 3 to 4 weeks in which to re-train.

In the Fed. Rep. of Germany, the report speaks of a course lasting 150 hours, which is felt to be insufficient. It is admitted, however, that many newly qualified teachers continue their training as they go along, while actually "on the job".

The French report refers to a "scheme ... whereby every established teacher in work is entitled to 36 weeks of full-time training to be spread over his career". This scheme, of course, will not be applied only to mathematics.

5.3.3 *Types of re-training. Training staff*

Working groups. In Upper Austria, about sixty teachers interested in the new mathematics teaching have undertaken, under the auspices of the Institute of Education of that State (the Institute is a re-training centre), to take the lead in organizing working parties as a means of initiating their colleagues into the reform.

In Switzerland, training teams have been set up, composed of primary school teachers who have been through the *écoles normales*, training staff and specialist upper-secondary mathematics masters.

Belgium refers to "study days", and notes that some re-training in that country takes place through correspondence courses, a system also used in France for the theoretical side of maths training for primary-school teachers.

The French report also points to the considerable effort towards teacher-training made by the schools radio and TV broadcasting service. The teaching sequences, under the title of "teaching workshop", are accompanied by documentary material to help the students. Video-tape recordings of the same sequences are much used in the *écoles normales*, where they give rise to fruitful discussion.

In Belgium, correspondence courses are combined with TV broadcasts co-produced by Radio-télévision belge and the Ministry of Education. Accompanying manuals are sent to anyone asking for them (30,000 in 1972). The primary aim of these broadcasts is to sensitise teachers to modern mathematics.

The Fed. Rep. of Germany speaks of "central courses", where the number of potential trainees creates a problem. If, on the other hand, "regional courses" are attempted, another problem arises, namely the *quality* of the "animateurs" or "multipliers".

The United Kingdom mentions the creation, as part of the project "Mathematics 5—13" (Nuffield Foundation), of "mathematics centres". "The teachers involved in the project were to come to these centres to learn more mathematics themselves, for refreshment and for discussion ... The centres vary from the non-physical centre, so that any organized meeting of teachers ... is deemed to be a teachers' centre, to those centres specially ... equipped with working areas." "It must be borne in mind that the ideas had usually arisen in schools, so the project was really a gathering, pruning and dissemination of ideas ... In many centres the teachers themselves now put forward the ideas and express their needs."

5.3.4 *Target public*

It is essentially the teachers who are aimed at. The French report mentions adaptation courses for lecturers at *écoles normales*, as well as courses for Inspectors of Education at the level of the *départements*, who have recently been required to take part in the initial and in-service training of teachers.

5.3.5 *Content of re-training courses*

Courses deal mainly with mathematics itself and how to teach it.

Austrian courses take place in three stages:

a) Familiarising the teachers with the materials that their pupils will use;

b) Study of the teaching problems raised by the reform;

c) Study of the theoretical concepts underlying the new maths syllabus.

Re-training courses have often been most influenced by experience gained in the establishments responsible for the initial training of teachers.

The Belgian correspondence course is drafted in the spirit of the provisional primary education curriculum published by the State (1971). It comprises mathematical contents and methodology cards (which never provide "model lessons").

In Switzerland, where action has often been in two stages (training of instructors and then, with their help, training of primary teachers), the content of the course comprises both mathematics and teaching theory.

The French report contributes a new element: it urges that what the teachers should be offered is not so much "teaching recipes" as the possibility of creating their own methods on the basis of collective work (group activities). It also stresses the importance of "research work", in particular at training staff level. Such work obliges tutors to polish up their knowledge and gain a thorough conception of the problems actually facing the children.

None of the reports makes any mention of psycho-pedagogical training, still less of psychological training in the cognitive, logico-mathematical field and in that of learning. May this be due to the fact that, as the accent was placed on re-training, it was felt best to deal with "first things first" by restricting discussion to the didactic aspect?

6. APPLICATION MEASURES

6.1 Experiments

The introduction into primary-school teaching of new mathematical ideas is so important an undertaking, and one involving so many risks, that no ministry of education has felt justified in generalizing it without prior recourse, over a relatively long period, to convincing and properly supervised experiments, conducted with the greatest care.

6.1.1 *Bodies entrusted with the experiments*

In *Austria*, the Centre for School Experiments and Development plans to carry out in-depth research into the methodology of new maths teaching. The teacher-training colleges, about to be transformed into universities of education, will be asked to co-operate in this work.

In *Belgium*, experimentation is being directed by the usual senior staff (head teachers and inspectors).

In French-speaking *Switzerland*, exploratory work of an experimental nature is mainly in the hands of cantonal education research centres.

In *France*, research teams have been working under the auspices of the Institut National de Recherche et de Documentation Pédagogiques (INRDP) and the Instituts de Recherche sur l'Enseignement Mathématique (IREM). The latter, which were instructed to give priority to helping in the application of the new curricula at secondary-school level, have none the less had an impact on primary-school teaching through the medium of *école normale* lecturers attending their courses.

6.1.2 *Experimental devices*

On this point the reports provide little information. Austria speaks of questionnaires and experimental tests. It refers to a "wide-ranging error-analysis" which should soon make it possible "to establish where individual difficulties of comprehension among pupils lie", and what "therapeutic measures" are needed. In this way it is hoped to establish, more precisely than at present, "minimal requirements for each school year".

Elsewhere there is reference to "experimental classes". Ireland speaks of 150 pilot schools in which are found teachers especially interested in the new teaching. in Belgium, about 30 experimental schools are active in the 5 French-speaking provinces. In particular, mention is made of the experiments made by F. Papy at the State primary school in the rue Berkendael (Brussels) and by L. Jeronnez at the Royal Atheneum (Ixelles).

6.2 **Generalisation**

6.2.1 *Progress of generalisation*

In *Austria*, the innovations are being introduced gradually so as to obtain the co-operation of teachers and parents.

In *Belgium*, there is no obligation to apply the new syllabus. For the 1973-74 school year, the figures are the following:

1st year	*2nd year*	*3rd year*	*4th year*
61	54	37	2 grades

The experiment, which began in 1971, will be extended at the rate proposed by the Ministry of Education and in those schools which accept the reform.

The syllabus is also being gradually generalised in *Switzerland.* Here are the statistics for 1973-74:

1st year: All the French-speaking and two German-speaking cantons have generalised the new maths teaching. Ten cantons have introduced it experimentally in a number of grades. The remainder, apparently, have not yet done anything.

2nd year: Generalisation in two cantons (one French- and one German-speaking). Experiments in 14 cantons.

3rd year: Generalisation in one canton (German-speaking). Experiments in 10 cantons.

4th year: (= secondary education in some cantons). Generalisation in two cantons (German-speaking). Experiments in 7 cantons.

The Swiss report notes that the only canton (German-speaking) to announce the generalisation of the new teaching in grades 1 to 4 uses, however, a syllabus and text-books which are still very close to the traditional method.

In the *Fed. Rep. of Germany*, the rate at which the new mathematics is being introduced into schools in the *Länder* may be shown as follows:

— 1st year	from 25 to 98 %
— 2nd year	from 17 to 80 %
— 3rd year	from 5 to 80 %
— 4th year	from 2 to 80 %

In *France*, an opinion poll conducted in 1972 gave the following information:

— 10 % of teachers feel that they have completely renewed their methods

— 40 % feel that they have come somewhere near to the proposed target

— 25 % have simply readjusted their traditional teaching methods

— 10 % did not reply

— the remaining 15 % are carrying on as before.

One teacher out of two feels "really concerned with reform".

The reformed syllabus for Ireland was published in 1971. No date has, however, been fixed for going over from the old to the new. "It was left to individual teachers to adopt the new syllabus or portions of it when they felt they were ready to do so." This highly liberal approach has had good results: "the response has on the whole been gratifying. The new syllabus has been welcomed by primary teachers in general, in spite of the initial fears of many of them . . . The philosophy of the new mathematics has been accepted."

6.2.2 *Special arrangements made at the time of generalisation*

These arrangements essentially concern the training (re-training) of teachers. On this point the reader is referred to what has been said above (see 5.3, page 38).

Let us also recall the fairly considerable effort made in the matter of educational manuals (see also 4.2.9, page 33). Ireland, in particular, mentions the publication in 1971 (when the new general curriculum was introduced) of a 700-page work, a "Teachers'

Handbook" for primary teachers, school managers and students in the colleges of education. The handbook devotes 150 pages to mathematics: syllabus, teaching method and new material.

6.3 Evaluation

Too much was at stake in introducing new mathematics in schools for the assessment of the results to be neglected. Furthermore, educational research and, more especially, the science of curricula (curriculum development, Curriculumforschung), have made sufficient progress during the last twenty years to allow the making of evaluation studies likely to be of direct help to those responsible for schools.

6.3.1 *Bodies responsible for evaluation*

These are nearly always research centres attached to the ministries of education. The example of French-speaking Switzerland is worth quoting:

The "Institut romand de recherches et de documentation pédagogiques" (IRDP) has been officially made responsible, on a permanent basis, for evaluation of the new teaching. An ad hoc liaison officer has been appointed for this purpose. In addition, a "Commission d'évaluation de la mathématique" (CEM) composed of school inspectors, secondary-school mathematics teachers, instructors *(monitrices)* in charge of the re-training of teachers, psychologists and educational research workers, has the task of advising and assisting the IRDP. The cantonal research centres co-operate ex officio in the evaluation work. As a start, questionnaires will be sent to teachers and tests given to pupils. Evaluation has a "training value" in the sense that it should serve to adapt all aspects of teaching: syllabuses, text-books, materials and teacher-training.

The German report states that 4 of the 11 *Länder*, having opted for the new syllabus, provide for supervision tests to be organized by specially appointed institutions:

— *Baden-Württemberg:* Institut für Bildungsplanung bei Modellschulen, Freiburg-im-Breisgau

— *Bavaria:* Institut für Schulpädagogik, Munich

— *Hessen:* Goethe-Universität, Frankfurt a/M.

— *North Rhine-Westphalia:* Lehrplankommission.

6.3.2 *Evaluation of pupils' progress*

Belgium admits that the absence of objective tests causes a feeling of insecurity among the teachers. A random check, by means of a test containing some 70 questions and covering the entire syllabus, was carried out among the 2nd-grade pupils, but at the end of 1973 the results were not yet known.

The testing of pupils in *French-speaking Switzerland* (cf. 6.3.1 above) implies the advance translation of the whole syllabus into "operational objectives" to ensure that the *major objectives* of the new teaching are not overlooked.

The report by *France* notes that 5 psychological and 8 mathematical tests were worked out. The results were expected in 1974. The same report makes various criticisms (it is the only one to do so in any detail):

— The connections between numerical and non-numerical activities are under-exploited.

— There is a premature tendency to formalize teaching. Manipulation and the concrete aspect, which should be the basis for "active" mathematics, are forgotten.

— A distinction is not drawn between addition, the law of internal composition, and the additive function.

— Arrangement of objects to bases other than 10 gives rise to abuse; after all, calculation to base 10 must be given priority.

— The links between the properties of the operations and operation techniques are badly explained.

— The examination for admission to the first year of the first secondary cycle *(sixième)* distorts working conditions in the preceding primary class *(cours moyen 2)* and the spirit that should prevail at this stage of the teaching process.

The *United Kingdom* report also mentions the tendency to premature symbolisation.

The *Irish* report is more positive. It considers that the exploration of the environment, positional notation systems (to bases other than 10), graphical work, etc., lead to "a new awareness" of mathematical processes with a happy effect on pupils' attitude to their own environment. The eagerness with which these same pupils seek to solve new problems seems to prove that they are more interested in mathematics than their elders were.

Some years ago already (1970-71) a comparative study was made in *Finland* between classes using the old and the new text-books respectively, but it failed to reveal any great differences.

"In traditional areas of elementary mathematics (arithmetic), there are no significant differences." In the new areas (logic), pupils still using the old books continue to get quite reasonable marks. This, it is thought, results from the fact that they were given information outside the text-books. The example is instructive. It shows that an innovation started at one point in the school system affects other points, by a form of symbiosis.

6.3.3 *Evaluation of teachers' contribution*

The reports are modest on this point. The *Iris* report writes: "Our greatest difficulty in introducing the new syllabus was our teachers' lack of knowledge and lack of appreciation of new mathematics . . . Primary teachers are general practitioners" who at present have to be re-trained in almost all subjects: social studies, arts and crafts, languages. Thus the whole question of their training is in the melting-pot.

In *Belgium* it is felt that the supervision of teachers is insufficient and it is hoped to obtain the assistance of regional "animateurs" selected from the teaching body and placed under the instructions of the inspectorate.

The *Swiss* report mentions three difficulties:

— Need to provide in-service training for a large number of teachers simultaneously.

The knowledge acquired over several years by experimental class instructors cannot be assimilated at any faster rate by teachers who do not have the same incentive as the enthusiastic teachers who were involved in the first experiments.

— Need to provide longer in-service training for teachers in the upper primary grades.

These should be better acquainted than their colleagues in the other grades with both the higher and lower stretches of the new syllabus.

— Need to keep in mind that the teacher is not a maths specialist. He must also keep abreast of the considerable changes that are taking place in other subjects as well.

6.3.4 *Teachers' evaluation of the new teaching*

About 200 Austrian teachers expressed the following opinions on the "new maths" experiments:

"— Are the children not too young to learn mathematics?

— Will the pupils need new maths in the secondary grammar schools?

— Will the pupils be able to make use of new maths in later life", especially those who will be learning a "practical" trade later on?

Certain teachers and psychologists in the Fed. Rep. of Germany think that the experimental basis of the new teaching is still too restricted and that the data relating to the formation of abstract concepts are lacking.

The extent to which French teachers are in agreement with the reform may be expressed as follows:

— 17 % fully approve;
— 40 % are inclined to approve;
— 31 % are inclined to disapprove;
— 12 % disapprove.

Amongst those who have made an effort to introduce the reform (one out of two; see 6.2.1 above), 30 % are not convinced that the aims of the new teaching are valid.

As regards the Irish teachers who have applied the experimental syllabus in whole or in part, they have expressed themselves satisfied with the results obtained.

6.3.5 *Mathematicians' evaluation of the new teaching*

Only the *German* report includes the opinion of the mathematicians. In doing so it echoes certain criticisms made in the course of 1973.

Mathematicians consider that it is illusory to claim that the theory of sets can be taught to children, since it is beyond their grasp. The instruction which the teachers think they are giving — particularly in view of the terminology employed — gives misleading ideas about mathematics.

"Scholars in the field of applied mathematics, scientists, technicians and economists, fear that students being educated by means of the new curriculum lack skill in arithmetic and simple algebraic techniques." The report notes, however, that "investigations upon this subject — although very few in number — do not support this opinion".

6.3.6 *Parents' evaluation*

In *Germany*, "parents feel uneasy at being unable to answer the questions of their children". This feeling has helped to foment recent press campaigns against the new maths teaching, and these have been backed by members of parliament and certain doctors "who claim to have diagnosed modern mathematics as the cause of stress symptoms in children".

6.3.7 *Recommendations resulting from the evaluations*

The *Austrian* report is explicit on this point. It is hoped that:

— "the connection between the various aspects of the new maths with the more recent results obtained from developmental psychology" can be better specified;

— "the relationship to the other areas of a comprehensive cognitive education" can be strengthened;

— "scientific modes of thought" can be introduced into "the area of primary-school teaching"; and that

— a general maths syllabus can be established which extends from nursery school to university.

7 CONCLUSION

In my introduction to the present text, I promised to give a "situation report". Now that I am about to lay down my pen, I observe that the kaleidoscopic reality I have been privileged to explore could brook no static description. It is, like life itself, pregnant with dynamism and diversity.

A few remarks only, then, by way of conclusion.

Let me begin by the viewpoint of the mathematicians, fifteen years after Royaumont. In some ways the seminar had unfolded on a note of triumph. The device flourished by Jean Dieudonné, "Euclid must go!", even appeared to be the signal for a holy war (1).

Royaumont helped to give respectability, in the 60s, to a conception of mathematics strongly inspired by the Bourbaki school and set forth in the "Chambéry Charter" (see p. 14 above): "It is the constructive, axiomatic and structural conception which characterises present-day mathematics" (1968).

Since then, other voices have been heard, such as that of Prof. René Thom (gold medallist of the 1958 International Mathematics Congress) of the *Institut des hautes études scientifiques*, Paris. In a paper read to the 2nd International Congress on mathematics teaching (Exeter, Sept. 1972), under the title "Mathématiques modernes et mathématiques de toujours" (modern mathematics and all-time mathematics), René Thom inveighed against excessive structuralism and pleaded for the rights of *meaning*, completeness and reality. To him, structures are only the surface of mathematics. They must constantly give way to the deeper structures of reality, themselves charged with divers significations.

In favour of the structuralism criticised by Thom, the argument that it had clarity was valid, but it was a somewhat anaemic clarity. The crying need was for a return to reality.

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(1) In justice to Professor Dieudonné, however, it should be recalled that immediately after shouting his battle-cry he hastened to pay a tribute to the Euclid he had just decried:

"If the whole programme I have in mind had to be summarised in one slogan it would be: **Euclid must go!** This statement may perhaps shock some of you, but I would like to show you in some detail the strong arguments in its favour. Let me first say that I have the deepest admiration for the achievements of the Greeks in mathematics: I consider their creation of geometry perhaps the most extraordinary intellectual accomplishment ever realised by mankind. It is thanks to the Greeks that we have been able to erect the towering structure of modern science."

From "New Thinking in School Mathematics", p. 35. Published by the Organisation for European Economic Co-operation, Office for Scientific and Technical Personnel, Paris, 1961.

This finds an echo in the thinking of Prof. Gonseth (Switzerland). All his life Gonseth has been evolving a "philosophy open to experience", with the emphasis on dialectics that must at all times link the "bookish theoric" — the fabrication of models and structures — with reality. Once apprehended by the dialectic approach, reality can then be explained and transformed.

A comforting "wisdom" results. Mathematics, which may, and in some circumstances must, be a pure working of the mind, is primarily, for all of us, an instrument of adaptation to reality. It is a tool, and, as such, gains acceptance.

Let us now turn towards the viewpoint of the teachers.

"Modern maths" has swept over them rather like a whirlwind. "Sets" invaded syllabi and handbooks: they also cluttered up the minds. After the first fine careless rapture, there seems to have been a return to a more reasoned approach. Such, at least, is the message of the reports from the eight countries consulted.

It is acknowledged that the "set theory" was not being taught to children. A theory of that kind is for mathematicians; it cannot be communicated, as such, to young minds.

But what can be done, using that theory as a basis, is to teach children to face their own reality and give them the means of organizing it. The task here is educative rather than instructive. At the controls we find, inescapably, the complex contemporary world with its many technologies. It is a task of prime importance, and it has, too, a social value. Hence we have doubtless done well to cease regarding the art of conducting the "four operations" as the duty of the primary school. But we have been well advised — and on this point the reports are categoric — not to throw out the baby with bath-water. Pupils who have been taught under the new syllabus "calculate" just as well as the others. It may even be thought that they calculate better, because they will have understood the internal workings of these operations — workings that are themselves founded on general properties into which the pupils have been initiated; and also because they will have been taught to make intelligent use of the electronic calculators which are flooding the market and which tomorrow will be found in every household and every classroom.

However, the keynote of the whole reform, at primary level, lies in helping the children, not so much to furnish their minds with "rudiments of mathematics", as to fortify their own thought. It is therefore proper to stress the close relation — spelt out in particular by the United Kingdom and Irish reports — between the new mathematics teaching and a "child-centred" education. Those educators, it is said, who were aiming to adapt themselves to the child and, to that end, studying the psychogenesis of intelligence as described by Piaget, have since been led to accept a large part of the recommendations of today's mathematicians. In any case, a child-centred education requires that account be taken of the child's own environment. And this leads to a warning note: let us go easy on the use of "materials", however sophisticated, in favour of the actual objects among which the child develops and over which he must, first and foremost, exercise his mastery.

The new teaching of mathematics at primary level consists, in the last analysis, of gradually helping each child, having regard to his talents, his needs, his environment, to adopt towards his own little world an attitude woven both of courage — a degree of combativity — and of logical intelligence.

Thus, once again, the "syllabus" is not the be-all and end-all of the school, and once again the role of the teacher is seen as fundamental. That, surely, is why all the reports are particularly insistent on this point: the reform will succeed only to the extent that the teachers can drench themselves in its spirit. True enough, that spirit has a mathematical component, but its essential component is psychological and quite simply human — a mixture of intelligence and sensitivity, rigour and tact, the lightness of air and the solidity of earth.

That spirit would seem to suffuse the eight countries that have sent in reports. Its invigorating presence has already borne fruit. But the fruit will be all the better for the encouragement of regular exchanges of views, like those started in Strasbourg in December 1973.

APPENDIX I

A CASE STUDY ON THE METHODOLOGY OF NEW MATHEMATICS TEACHING IN THE PRIMARY SCHOOL IN ENGLAND

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PREFACE

I would like to thank Dr. Semple, Chief Education Officer for Bexley (until December 1974) for allowing me to use Primary schools in Bexley for this case study; Mr. Tovey, Mathematics and Science Adviser (now Senior Education Officer for Further Education) and Miss Beswick, Primary Adviser for their help in selecting the schools and arranging with the Heads for my visits; the Heads of the schools visited and their staffs for their interest and co-operation; and Dr. Margaret Baron of Stockwell College for her valuable criticism and encouragement.

I. INTRODUCTION

School Buildings

There is no reason to suppose that the London Borough of Bexley is not typical of most authorities in England and that its schools are not typical of most schools in England. The school buildings varied enormously in style and facilities. Six schools were in pre-war buildings, the earliest being built in 1928. The latest built one was in 1964. Four schools had several permanent hutted classrooms; in one school scattered, on a sloping site over quite a wide area. Two had had extra classrooms built and one had a mobile hut in the school grounds temporarily to combat increase in numbers, but others had no additional buildings – one built for 240 children in 1936 now holds up to 370.

In most of the schools, the classrooms were side by side opening off a corridor or hutted and spread about the site. All the schools were built using the idea of one teacher, one class, one classroom. In many cases, the corridor was wide enough only to provide a passage to the next room. In one or two schools, however, the corridor was just wide enough to provide a small working area. Here children could spread themselves out a little more. Children were found painting large pictures which consisted of geometrical plane figures arranged in pattern formation. In another class, because there was no space for water in the classroom, the teacher set up a water trough in the corridor and did work on measuring the capacity of various containers. A third group had made a study of the shapes to be found around the school. This work they displayed outside their classroom to give all the school an opportunity to see it. Cloakrooms, a hallway, a landing outside classrooms were also utilised in the same way.

Classrooms are usually rectangular. Many had an alcove intended originally as a store and shelved accordingly. It was also used as a quiet corner for reading or for using a class reference book, for practical work, for work left unfinished over a period of time or for material being collected for display later. It was usually a welcome addition to space in the classroom and to space required to do mathematical work.

Often it is difficult to separate mathematics from the rest of the curriculum and to distinguish what is mathematics and what is not. Early work on the measures connected with time, length, volume and capacity, weight, could well be thought of as science. And work on investigating the school site might come under geographical studies rather than mathematics.

One school mentioned the possibility that the walls between the alcoves might be knocked down to provide a large double room for two classes. Physical restrictions made compromises necessary, and teachers were not always free to work as they wished.

Organisation of the School

The children may enter a horizontally grouped school where the children in any one class are of roughly the same age, e.g. all the children will be approximately 5 years old or 8 years old. Or they may enter a vertically grouped school where each class contains children in a wide age range, e.g. the class may contain children in the 5 to 7 age range. Or they may enter a school where a mixture of the two is in operation. One of the four Infant schools was vertically grouped. One of the Infant departments of a Junior Mixed and Infant school had 4 vertically grouped classes, a further class contained children of 5 to 6, and the remaining three classes contained 6 and 7 year olds – the number of children not being sufficient, in fact, to have vertical grouping throughout. These arrangements are chosen by the Head and staff and may change if the Head changes, if there are considerable staff changes or indeed if it appears to be an unsatisfactory arrangement. Schools were constantly reviewing their patterns of working.

The smallest Infant school had 130 children and 5 classes. Three schools contained over 500 children (one Junior and two Junior Mixed and Infant – the largest with 570 children and 17 classes). The number of children in a class ranged from about 26 children to about 40. Where a class contained a wider age range there was a smaller number of children. But most schools had more teachers than classes – some of these additional teachers, without responsibility for a class were full time, but many were part time. In two schools, the Deputy Head was without a class – one helped with probationary teachers and administration. Her Head made herself responsible for the mathematics. These classless teachers, however, usually took groups of children – for remedial reading, to give additional reading and language work for the older infants (7 year olds).

Contact between Schools

Where there are separate Infant and Junior schools, Heads are in contact with each other, but the other teachers may have little official contact. Heads may agree on certain things such as the importance of practical work in mathematics, but, if they disagreed on something and each felt strongly about the situation, they would not change things. Sometimes, when children are about to go up to the

Junior school, they are taken over to the new school to look round, and their new teacher comes to the Infant school to meet them and to find out what sort of work the children have been doing, particularly in mathematics. Often, however, there is no opportunity for the Infant teacher to see what the children are doing when they get to the Junior school. Even though the same scheme of work e.g. "Fletcher" (Appendix II) is used, methods may be totally different – in the Infant school, the teacher may have given the children individual work, and in the Junior school there may be class work. The idea of the Junior Mixed and Infant school under the one Head was favoured – when children are under the same roof, there is continuity – this is important in mathematics.

There is less contact between the Junior and the Secondary school and much disagreement on the methods of teaching mathematics. Children from several Junior schools feed any one Secondary school. While the Junior children usually visit their new Secondary school for a day, it is only infrequently that a Primary teacher visits a Secondary school or a Secondary teacher a Primary school. This difficulty of establishing effective contact between Primary and Secondary teachers often leads to a failure to understand or tackle common problems. Primary teachers lack an adequate knowledge of aim and purpose in secondary mathematics teaching, and secondary teachers fail to appreciate the methodological approach in the Primary school.

Staff Stability

Most of the schools were lucky enough to have reasonable stability of staff over the last three or four years. One or two schools had had no staff changes for two years. This can have nothing but a good effect on the teaching of mathematics where continuity is so important. All schools had appointed at least one probationary teacher on the staff; teaching mathematics often proved more difficult for them than other aspects of the primary curriculum. Usually the Head looks after the probationary teacher and was prepared to give as much help as possible in areas of the curriculum where that probationary teacher might have difficulty.

Mathematics Courses and the Staff

Many teachers attended mathematics courses or conferences. These ranged from one-day courses and evening courses organised by Local Education Authorities (LEAs) and given by mathematics lecturers, mathematics teachers and others interested in the subject, a two-day course at a College of Education, given by members of the

Mathematics Department, Association of Teachers of Mathematics meetings (evenings, Saturdays or the Annual Conference – residential and lasting several days), three-day Department of Education and Science (DES) mathematics courses run for the LEA by Her Majesty's Inspector and follow-ups, to a one-term full time (11 weeks) DES mathematics course at a College of Education (an Infant Head took this course). A number of the teachers, including two Heads, had acted as tutors on DES mathematics courses and also gave mathematics courses organised by the Borough. A third Head was Chairman of the Working Party preparing the mathematical Guidelines for the Borough. Another teacher was also a member of this Working Party.

Contact between the School and Parents

Several schools have Parent Teacher or Parent Associations. They are regarded as friends of the school. One of their aims is to raise additional funds for the school to provide extra amenities — by organising a Christmas Fair perhaps – a swimming pool, a hand calculating machine were two examples. The parents also do jobs about the school such as putting up extra shelving, covering books. One school with no such official organisation had a school voluntary fund. These special groups also organise talks and discussions about various aspects of the curriculum. Some schools had special evenings or open days where children's work was displayed. The parents could see the children working and the teachers were available to talk about the individual child to the parents. Two schools had organised workshops for parents so that they could try out for themselves some of the mathematics their children were doing. Many parents worried about it being different from what they had done – "sums" and "tables" were mentioned. Most schools found the majority of parents to be interested and co-operative.

II. THE USE OF EXPERIENCE AND PLAY FOR YOUNG CHILDREN

Reception

Children arrive in the Infant school (normally at 5 +) with varying backgrounds and experience. Whilst some of them have good speech and vocabulary and are well informed and socially adjusted, others come from largely uncultured backgrounds and are both inarticulate and innumerate.

Many children cannot verbalise until they begin to read and, when they are unable to understand such questions as "how" and "why" there are extreme problems. Interviewing parents when they arrived to enter their children for school, an Infant school Headmistress was asked to advise on how the children might be prepared for admission. She replied, "Talk to your children, encourage them to draw pictures, and to develop an interest in words; encourage them to be number conscious; buy a pack of cards and games like Snakes and Ladders and Dominoes and play these games with the children." The Infant school must compensate for what is missing in pre-school experience. In the classroom, a wide selection of materials, equipment and activities is available for the children to handle and with which to experiment. The children's choice of activities and their "talk" with other children give the teacher a clue as to the stage of development reached and to their intuitive ideas on number, space and time.

Materials

If children enter the Infant school after previous attendance at a Nursery or Play school, they will be familiar with much that happens in their new situation. If they come into school straight from home, they may find things very bewildering. The natural environment, man-made materials and play equipment provide most of the opportunities for mathematical development. Typical objects from the natural environment commonly in use are water, sand, earth, gravel, small pebbles, stones, shells, fir-cones and conkers (Spanish chestnuts).

Many of the man-made items started life in the home and would have been thrown away if they had not been collected for use in school – they are likely to be found in the junk box in the classroom: – cardboard and paper cartons, toilet roll holders and other tubes of varying sizes, cotton reels (wooden and plastic), metal bottle tops, corks. In addition, there are such objects as metal washers, buttons, screws, beads (both round and square), cocktail sticks, matchsticks (often purchased without striking heads), plasticene, clay, marbles, coins, pegs, counters, geometrical shapes cut from card or polystyrene. Food which has good storage quality is also much in evidence — peas, beans, lentils, pearl barley, flour, rice, dog biscuits. No classroom is complete without a Wendy House or home corner, weighing balances and a small cooker. Different kinds of shops are opened from time to time – post office, grocery and sweet shops are among the favourites. Many small toys such as model cars, farm animals and all kinds of assorted shapes are constantly in use.

Teachers' own games made at a particular point for special purposes are frequently used, in addition to commercially-produced games. Some of these are familiar from home — Snakes and Ladders, Ludo, Dominoes, Stepalong, Connect, Number Lotto. Others have been purposely constructed for mathematical play, e.g. Mr Money, shape matching and many dice games are used to provide particular number experiences. Epps and Deans "Mathematical Games" was published as a book, the pages of which can be separated to form individual games (Appendix II). Packs of cards find many different uses and constructional toys provide experience of shape. For example, jigsaws, mosaic shapes, tangram pieces, pegboard and pegs, pattern blocks, provide experience in two dimensions — whilst Playpax, Lego, Meccano, Fit-Bits and sets of large bricks are used for exploring three-dimensional space. Structural apparatus is normally introduced as play material, e.g. Cuisenaire Rods, Unifix Cubes and Dienes Logiblocs.

Water

Many classes are fortunate enough to have a sink in the classroom. The child turns on the tap and watches the water flowing into the sink and filling it. He watches the water running away when the plug is pulled out. Children in other classes must make do with a water trough. Due to pressure of space in the classoom, this can, in some cases, only be in use at specified times. Most children are attracted to water, and initially the teacher allows the child simply to feel the water trickling over his hand and watching the movement. Later, the child contrasts this with the sensations he gets from other fluids, perhaps grain and more often sand. The teacher adds to the enjoyment and excitement, as well as providing a new learning situation and the means to develop it, by removing equipment and providing new material from time to time. With a funnel, the water can be channelled into containers of varying sizes and shapes — or simply back into the trough. Empty plastic squeezy bottles provide a useful source of containers. Because they can be cut and pierced in various ways, they are extremely versatile. Watching water pouring from a container punctured with holes at different levels provides an interesting contrast from water running from a tap — patterns can be made and developed. A variety of containers of different shapes and sizes — tall ones, short ones, wide and narrow — helps the child to develop new skills. He learns to pour from one to another without spilling, he fills a container to a marked level, he estimates and checks whether a particular container holds more or less than each of the others.

The Role of the Teacher

Whilst the teacher puts no restrictions on this free play except by her selection of utensils, she watches the child in action, encouraging him to describe what he sees and to say, in advance, what he thinks will happen. She introduces new vocabulary appropriate to the situation and by her questions and comments she stimulates the child's thinking into new channels. One day the child plays with water; the next day he investigates the sand trough. He repeats many of the activities with sand which he has already carried out with water. The teacher encourages him to note the similarity and also the differences in the properties of the two materials. When he plays with dry sand, it behaves much as the water but, by contrast, when the sand is wet, the funnel is not much use – neither are the squeezy bottles. The child finds the sand sticks to him and sticks in lumps — he can make shapes with it which need no container to hold them — he can write and make patterns in the sand.

Through play, the child is learning about the properties of materials, how they feel and how they move. He is making natural classifications which are much more real to him than those achieved by using structural apparatus.

Weighing

Early experiences with sand and water can be used to get the feel of a balance. How do two identical containers, one filled with wet sand and one of dry sand, behave when placed on a balance? And can we compare them with other objects and materials? It is satisfying to see one side of the balance dip rapidly whilst the other side rises at an equal speed. The weighing part of the home corner usually contains appropriate materials and equipment. In most cases, the store probably occupies only one small table, or possibly two pushed together, but labelled to show its purpose. "Balancing here — only two children", "Two children may weigh here", or simply "Weight". Many of the materials already listed are used together with packages made up to look like goods in a real shop. The goods are placed in labelled jars and in open cartons and scoops and spoons are provided for ease of handling. For measuring out specified quantities, the child finds tea and medicine spoons, cups, jars, yoghurt and cream pots, empty packets and boxes. Balances with identical scale pans come in various patterns. Some have hanging pans which may be as small as saucers or as big as buckets or washing-up bowls. Others have supported pans, about the same size as household scales. Occasionally a spring balance, hand held, may be used.

The child "balancing" for the first time spends much effort in putting objects and materials on either side or both sides completely at random. Through the teacher's questions and comments, he becomes more selective in his choice of materials and more careful in his measuring out of amounts. In most cases, these activities are linked with cooking — making biscuits and small cakes, sometimes even baking bread. Through cooking activities, the child becomes very much aware of the need for precision in measuring. In selling the products afterwards, inequalities of size are immediately apparent. Play becomes real life when coins of the realm are handed over in exchange for cakes or biscuits.

Shopping

Many young children have practical shopping experiences, some may even have been shopping alone. Others are less fortunate and may hardly have been inside a shop. The class shop takes many forms, but it does enable the children to act out real situations which they may or may not have previously encountered. The materials for the shop may be empty packages and tins collected from home for a grocery or sweet shop. They may be objects which the children have made themselves, such as fruit or sweets, from clay or plasticine. The post office is popular with some items made and some bought in a "Post office game". Above all, the shop is a social activity, and this social activity is a necessary background to the development of any mathematical concepts which can arise out of the situation; the children are very much concerned with roles, in which one acts as the shop-keeper, others as customers; goods are purchased and wrapped, money changes hands, change is given and conversation takes place about the size, the rising costs and so forth.

The Wendy House

The Wendy House too brings home directly into the classroom. It may be purpose-built — four walls, windows in appropriate places, a door and a roof – but house is often too grand a name. It may often be two pieces of wood, one piece with a window, one piece with a door, hinged together to form the side of a house. Or it may be just two walls made from scrap material stuck together and painted over, or a clothes horse covered with fabric. Furnishings vary – one or two small chairs, perhaps a bed, some sort of table with cutlery and crockery, probably some dolls or teddy bears. Having a tea party, matching cups and saucers, knives and forks to plates, making sure there is enough for everyone are natural ways of developing concepts of equivalent sets and one-to-one correspondence. Dressing and undressing dolls and teddies, doing up buttons, provide useful further experience. Filling the teapot and ensuring that there is enough tea to fill all the cups reinforces the skills acquired at the water trough and carries these skills into a new situation.

Model-making

In model-making, the child gains further experience of relating the classroom activities to real life and through it, his powers of observation can be developed much further. Junk cartons and boxes of all shapes and sizes can be glued together to make objects seen in the home, in the streets, e.g. furniture, cupboards, the TV set, shelves, lorries, buses, a pillar box, street lamps, buildings and animals. Selecting the most suitable containers, knowing the reason for his choice, learning why a particular shape is unsuitable and why a certain shape is too small — all of these call the child's judgement into action and develop his ideas on shape and size. Once the model has been put together it is covered with paint or paper; discussions about the amount of paper or paint required raise new ideas about covering surfaces, and these discussions are not restricted to flat surfaces in two dimensions. Questions arise concerning the size of the model in relation to the size of the actual object. The models themselves can be ordered in size; for example, making a model of the local street might involve shops, pillar boxes and telephone kiosks. Furnishing a doll's house with furniture fashioned to fit into the rooms is a further example of the development, through play activities, of appropriate concepts of scale. Making animals for a farm or zoo certainly involves ordering of size. Besides the physical activities involved, with the development of manual skills in the handling of material, conversation between teacher and child shows how much of all this material the child can relate to his own personal experiences, whether real and derived from visits to the zoo or the farm, or secondhand and absorbed from watching TV.

Games

The experiences and play activities mentioned up to now have been related to the material world and the child's own environment; in these activities his imagination, and his capacity to copy other people's actions and words have been the prime factors in the situation. Games do, in fact, provide a further link with home in general, although there are many children who have little experience of these before they come to school. Board games which rely on dice with clear rules for the movement of play are very popular. These are competitive games — played with one or two children only; the game moves quite quickly, the next turn comes soon, and each child takes an active interest in the other children's moves. Counting out the correct number of steps according to the number of spots on the face of the dice is the central activity. The winner is the one who was most fortunate in throwing higher numbers than the others. There are innumerable variations in the basic game in which each person makes a single move. If each child has several counters, a choice of moves may be possible, so skill in choosing the most suitable move means luck in getting higher scores is relatively less important. Handicaps and rewards may be involved when landing on a particular spot. "Move two forwards", "move three backwards", may be the instruction. No skills, just luck is involved again!

The imaginative teacher has often contributed her own game — perhaps in relation with a story in which the children became interested. "The Three Little Pigs" provides situations which can be translated into a race game, with handicaps and rewards. Instructions are often written on the board itself, or they may be issued on cards, each player in turn taking a card and placing it on the bottom of the pile, after use. "Go back 4, the wolf is waiting!", or, "Start again, the wolf has blown you back to the beginning" — instructions of this kind bring the story to life and, when used in conjunction with *acting* the story and writing about it, making models and so on, add a new dimension. When the teacher invents her own game, she is able to gear it to the interests and experiences of the children.

"Domino type" games enjoy great popularity and there are many versions. Instead of just matching spots against like spots, spots may be matched against numerals and numerals against number words. Shape may be matched against shape and colour against colour. In this way, all kinds of learning are facilitated and there are many commercial domino games as well as the teacher's own. *"Connect"* matches lines of up to three colours, edge against edge, providing visual experience of rotations and reflections. Most cards are square, or 2 x 1 rectangles, but triangles and even hexagonal cards are sometimes used. "Ten up" cards are triangular and in matching triangles, the sum must be 10. "Mathematical Games" are race games, again constructed with particular mathematical ideas in mind which the inventors have elaborated.

Many games involve pattern-making involving matching colour as well as shape. The child may well get his first ideas of symmetry from some such activity. Jigsaws provide useful experience in positioning pieces, even though the pieces themselves may not be any particular shape. Pictures are often made from card or paper shapes, glued down to make a permanent recording. A shape cut up into several pieces can be re-arranged to make a new shape. Ideas about conservation of area begin to develop — all the pieces have been used in the picture. Children make "collage" pictures. This new activity is different in that the pieces must not overlap, but the surface must be covered.

Constructional Activities

Even with thin pieces, many children build vertically in addition to spreading out over a surface so it is not surprising that there are many types of materials which cater for this more successfully. Bricks made from light polythene but as large as house bricks can be stacked vertically to make a more secure structure. On a smaller scale, "Lego" bricks behave in the same way. The child learns to make a strong wall by overlapping courses. Cuisenaire rods can also be used for building, although the structure is less secure. The child explores possible alternatives when he has run out of a particular size. He gradually develops confidence in handling and assembling the material. Coloured rods and bricks help to familiarise him with ways of *naming* pieces. He learns that he can rely on any red piece to replace any other red piece. Logiblocs are also used for building; children sort them, play domino-type matching games with them — sometimes alone and sometimes in competition with others.

Boys frequently get great satisfaction out of making models with constructional toys; girls are not disinterested, but frequently their preferences and enthusiasms lie elsewhere. "Fit-bits" and "Meccano" are used a great deal in schools, the former being a larger version in wood of the latter which can be plastic or metal. Skills developed include turning nuts and bolts, fitting pieces together, making symmetrical shapes, joining two short pieces to make long ones if not enough are available. Some children make recognisable models of bridges, trucks, cranes, etc., whilst others use ideas gained from TV programmes or children's comics in making imaginative constructions for the explorations of space. In discussions, intuitive ideas of scale and purpose can be developed. The child becomes thoroughly absorbed in his imaginative constructs, often sharing his ideas not only with his friends, but also with all his class mates.

Summary

The use of experience and play is regarded as a vital factor in the development of the young child. From birth, he has been engaged in exploring his immediate environment in one way or another. Each child, on entering school, has a background of experience which differs from every other child. The teacher offers a variety of situations which enable him to explore, to widen, to deepen his experience and to enlarge his understanding of himself, his class mates, his family, his friends and the world around him. Mathematics will play a major role in all this, for it will eventually enable him to sort, to classify and to quantify. To do this easily and successfully, the child must grow up feeling that mathematics is a part of his life and of the world around him. The teacher must also share these feelings, for only if she does so can she appreciate the child's excitement and wonder and be aware of situations which have mathematical potential. Besides providing materials and activities for this purpose, she will use natural situations which arise from day to day in the classroom. For instance, at "milk time", putting a straw into each bottle, giving one bottle to each child, etc.

The aim has been to show that the free play, which is so important and necessary to the young child, is being channelled into profitable lines and used as a broad basis for the child's development in understanding. All the experiences and play activities combine to give the child an awareness of many aspects of the world around him. Unless the teacher has a thorough grasp of basic mathematical ideas and their application in the environment of the classroom, she will be unable to exploit these situations to develop the child's mathematical knowledge. The child may be in a learning situation, but, if the teacher fails to appreciate the potential of that situation, the child will not learn.

III. INDIVIDUALISED AND GROUP LEARNING

Time spent on Mathematics

It is generally accepted in both the Infant and the Junior school in England that children work in a variety of ways, sometimes individually, sometimes in groups which may be as large as the whole class at times. The same is true in mathematics. In the Infant school, it is only occasionally that the whole class is doing mathematics at any given time. In the Junior school, it is rather the reverse. Mostly, all the children in the class were doing some mathematics at any given time, although it was often on different aspects. A great variety of forms of organisation was seen. Not only does the situation vary from school to school, it varies from class to class within a school. Teachers are usually left free to organise their day and teaching as they wish.

Traditionally morning, and preferably the time before playtime (the middle of the morning) has been thought of as the best time to do mathematics. In all the schools, it was unusual if some mathematics was not done every day, but the time spent on it varied from day to day. In the week, the children might spend as little as two hours or as much as five hours on their mathematics. The younger children usually spent less time, although if a child was really involved in the activity or chose to do mathematics in free-choice time, he could spend considerably longer than this.

Organisation of the Class

All the classes contained children of mixed ability – sometimes a wide spread. Teachers tried to solve the problem this presented in several ways. In some classes, the children are given a list of tasks to be done during a specified time. This is one way of working the "integrated day" – at any time during the day the class will be working on a mixture of subjects. The child chooses in what order he will do these tasks and has a specified length of time in which to complete his programme. After he has completed it, he can choose freely what he will next do. In this situation, the children will probably work individually, although they may link up for some activities if they are working on the same item. The teacher sometimes indicates that the child should work alone, e. g. he might wish to check that the child has grasped how to use the balance scale or whether he can perform simple calculations. On other occasions, children can choose for themselves whether to work together – in a measuring activity it may be more convenient to have another person holding the end of the tape measure or recording the measurement. Working the "integrated day" means that children at the same table may be working on entirely different things. They can choose to sit with their friends if they wish.

Others teachers prefer that a common activity should be going on at any group of tables. The children may be grouped in several ways. Some teachers prefer children with roughly the same ability or children who have reached the same stage to be working together. Other teachers prefer friendship or interest groups, but no teacher was rigid in sticking to just one way. As the activity changed during the day, so might the composition of the group – a child finishing his number work leaves the group to join a second group where writing the day's news is in progress.

To start off a mathematical topic, the teacher talks to all the group at a table, probably giving them the same task to complete. Then she moves on to another group. The children (even if they are of similar ability) work at different speeds. When they complete a task, the teacher gives them a new one until there is a general change of activity or the child may have a free choice. There may be work that is unfinished from a previous day – the teacher may wish to give him extra practice.

In another school, part of the day's programme is divided into three main activities and the class into three groups – one group does mathematics, a second group does English and the third group does painting, making models, etc. The teacher indicates which group each child is in and when the group shall change its activity. During the remainder of the day, these activities are completed and the other parts of the curriculum are followed. Teachers in both the Infant and the Junior school worked in this way.

In one Junior school, the children were "setted" across the year group into ability groups for mathematics, which took place at the same time each day for the whole year group. During this period, the Head and part-time extra teachers, as well as the class teachers, each took a group of children. Some of the teachers do not fully appreciate the advantage of a "team teaching" situation of this kind and would prefer to have their classes to themselves. All the other work was done in class groups, and this might include more mathematical activity arising out of projects.

Organisation in many Junior classes is of a more formal nature. Many Junior teachers favour class lessons. A particular idea, e. g. addition of fractions, is introduced by the teacher to the whole class. There is discussion with the children – they, as well as the teacher, put forward suggestions; the teacher asks questions, the children give answers; the child asks a question, the teacher answers, perhaps by a question. Some children do not actively join in the discussion, although this does not mean that they do not follow it. The children then work on material connected with this idea – practice of skills needed to effect the additions, exploratory questions that will test understanding of the technique – they may work in a group or individually. Some children will complete more than others of the material given – by observing what they do the teacher judges when to move on to the next idea. If some children complete all this material, they may do extra or harder questions on the same idea until other children have had a chance to complete an acceptable amount of work. The teacher then introduces the next idea, which may be a development from the previous one or may be something totally different and the process is repeated again.

Other teachers may start the class off together on a new mathematical activity, e. g. symmetry. Because of the nature of the idea, they may then share out the work involved. Different groups of children will work on different aspects of the topic – some on axial symmetry, others on rotational symmetry; some making pictures which have the required symmetry, others finding objects to satisfy their criteria. Then the class will come together again at the end so that groups can compare notes. Probably a display of the material collected will be mounted so that a complete picture is presented and all children can see the whole. This approach is particularly useful when a mathematical concept can be strengthened by exploring a variety of cases. For example, the children might make a collection of triangles found in the environment. Sometimes, the concept can be clarified and enlarged by developing a selection of different results and bringing them together. An example of this is a dice-tossing activity in which the whole class joins so that the results obtained by different groups can be compared and contrasted. One happy effect might be that children who were not involved in some aspect of the activity develop an interest in investigating it for themselves.

In a number of classes, different types of mathematics were going on, on different days. During one week, one class of children used their basic series in two mathematical lessons, did topic work on a third day and did practical measuring activities on the remaining two days. Children in another class, on one day, were working on logiblocs, computer-sorting cards, BBC "Mathematics Workshop" cards on "Grids", symmetry, doing multiplication practice, decorating their "Thoughtful files" with mathematical patterns.

When children change from an Infant school to a Junior school, methods of working in mathematics may be very different. This often presents problems – a child used to discussing what mathematics he is to do directly with his teacher can flounder when he finds himself in a formal class-teaching situation. Teachers are aware that there are problems and try to make the transition as smooth and as easy as possible for the children, but they are not always prepared to change the way they work in order to lessen these difficulties.

Layout of Classrooms

Desks are very seldom seen in schools these days. Children work at tables, on the floor, or at a wall perhaps. They keep their personal books etc. in drawers, which may be under the table or in a unit specially made for that purpose. In some Junior schools, the tables are kept very nearly in rows so that children face the front of the class. The blackboard is likely to be used for working out mathematical examples when the teacher has class lessons, but, in the other cases, it will probably just be used to indicate the tasks of the various groups or for notices. In most Infant schools, there is no blackboard visible; in some classrooms, there may not be one at all or it may be behind a mass of covering and be used as an additional display surface. In other classes, the tables are grouped round the room, perhaps two together, perhaps three or four. The tables are mostly rectangular (in two-by-one proportion), sometimes circular, occasionally half-octagonal. There is a chair for every child in the class, but he may not have a particular place. Often the children move from one table to the next as they change their activity. If the children are reading, they may sit at a table or they may go to a reading corner where a carpet is likely to be provided for the children to sit on. When the teacher talks to the whole class, she may gather the children together on the carpet or they may bring their chairs to a specific point in the room.

Mathematics Equipment and Materials in the Classroom

The smooth running of the class depends on much of the materials and equipment that is used being easily accessible to the children. Many classrooms have low cupboards and shelving round the room, the top of which can provide flat display space for junk models, e. g. for a railway engine, a church, a model of a garage forecourt or a street, etc. (the contents of the cupboards etc. were occasionally hidden behind doors or curtains, giving the room a tidier appearance but making access to materials less easy). Children are encouraged and usually required to select their own material and equipment and return it to its proper place after use. Typical examples are squared and plain paper and cards of varying sizes (widely used for mathematical work, e. g. for number patterns, geometrical models), work cards stored in pockets pinned to the wall or stacked in files, tape measures, rulers, trundle wheels, string, gummed strips or geometrical shapes, weighing apparatus, wooden cubes – one centimetre and two centimetre sizes, cartons of Unifix cubes, dice, etc. Much of the apparatus is stored in green plastic gardening trays, easily stacked when not in use. There are many other types of boxes and trays, which are colourful and stack easily – some specially constructed for the classroom but many in general use. Another example is stacking kitchen vegetable racks used for storing weights, cubes, etc.

Some equipment is kept in a resource area in a central position in the school to which all the classes have access. It might be additional sets of materials e. g. more Dienes Multibase arithmetic blocks, more spring balances and balancing scales; more expensive materials e. g. surveyor's tapes, board games and occasionally a single piece of equipment for use by the whole school e. g. personal scales. This store again might consist of shelves, cupboards or a trolley which can be wheeled into the classroom. The material required would be taken to the classroom and returned to the resource area when the task had been completed and it was no longer required. Inevitably, a piece of apparatus may be wanted in two classes at once or it may not be returned although it is no longer in use.

One teacher on the staff is probably responsible for keeping an eye on the situation and solving any problems which arise. That person is likely to have a responsibility allowance for mathematics. Many teachers would prefer to have all the material that is available distributed among the classes. They would then borrow from another teacher any additional material they wanted e. g. pairs of scales, this arrangement being more likely to ensure that the apparatus is returned promptly and in good condition. Some teachers may have a special piece of equipment in their classroom that they are trying out e. g. a hand calculating machine had been borrowed with a view to a possible purchase by the school if it was thought that it would be helpful and useful.

Not so much of any one particular type of apparatus is needed if not all the children in the class are doing the same mathematics at any one time. Some schools, too, felt that it was important to have a small quantity of good quality apparatus e. g. fibreglass surveyor's tapes, sensitive balancing scales, rather than a larger amount of cheaper material. Schools can spend their allowance of money as they wish. Some may concentrate on one aspect of the curriculum each year and spend the bulk of their money on this, others may share out the money fairly evenly so that each area of study gets some each year. This may enable the school to stock up on apparatus or books if a new scheme or new way of working is introduced e. g. buying sets of Dienes Multibase arithmetic blocks or sets of "Fletcher". Many teachers make their own board games and apparatus, e. g. bingo-type games for practising multiplication facts or for recognition of coins, matching shape games, dice games; nail boards, abaci, number lines.

Many classrooms, more in Infant schools than in Junior schools, have cupboards and shelves placed so as to create divisions within the room. Book-cases, wooden stands or simply a piece of hardboard or pegboard placed between two desks or tables to keep it upright were also used for this purpose. And, in one school, large rolls of corrugated paper, standing on their ends, served the same purpose. Display space is often at a premium, and these divisions helped to create more. In hutted classrooms particularly, there may be windows on two sides of the room. Frequent use is made of windows for mathematical display, e. g. outlines of geometrical plane figures, geometrical patterns, particularly symmetrical ones, pictures created from tangrams. Many rooms are high, frequently in older buildings, and if the display of mathematical work is to be accessible to the children, the top part of the walls cannot be used. However, in an occasional classroom, work was suspended from the ceiling. Some classrooms had beams to strengthen the structure, in other cases, the teacher had managed to stretch string across the room to provide hanging support. Geometrical models can be displayed to particular advantage in this way, also models of symmetrical objects – birds, butterflies, kites etc.

A child having his mathematical work displayed can feel a sense of achievement whether it is put up on the wall or whether the teacher, with the child, talks to the rest of the class about it. A teacher noticing that two seven year olds were colouring in patterns on squared paper, suggested they try making triangles from the squares by drawing in one diagonal of each square. By colouring in appropriate triangles, they found an example illustrating Pythagoras' theorem and went on to check that the "3, 4, 5" triangle fitted into this situation too. Although other children could not develop this work further, they were interested to look for other types of patterns by having seen this and discussing it with the teacher. A small group of six-and-a-half year olds had measured the lengths of their hands and feet. Discussing the results the class found that the child with the longest foot did not have the longest hand. Two children, not concerned with the original work, then measured the hands and feet of all the children in the class, and of some in other classes too, to test the incidence of this. This is an instance of how displaying and discussing work can provide a basis for further work.

The divisions and spaces created by partitions make it possible to have areas in the room devoted to different aspects of the curriculum, particularly practical ones – one area will certainly be devoted to mathematics.

London Borough of Bexley Guidelines

A Committee of Heads and teachers, with the Advisers, has been working together to produce "Guidelines" to help the Primary schools in teaching mathematics, for about two years, meeting fortnightly. So far, suggestions for work on Shape, Money and Time have been produced. For each of these topics considered, there are suggestions for vocabulary, activity and equipment – a progression of work through the Primary school is given. When these guidelines are finished, they will be published in sections so that any section can be revised if thought necessary at a later date. All Primary schools in the Borough will be issued with a copy. They will be encouraged to make use of it, but there will be no compulsion to do so.

"Guidelines" are intended to give guidance to the teacher on what mathematics she should be doing with her class. They are an outline only, and, whilst giving suggestions of activities which are suitable for use with a particular idea, it is left to the teacher to work out how to put them into practice in the classroom. It is also possible to use some of the suggestions and not others. A scheme gives much more detail and shows more fully how ideas can be put into practice in the classroom. It is usually expected that a scheme will be followed closely. This enables another teacher to get a clear idea of what mathematics a child has done.

Most Education Authorities in the country have produced guidelines or schemes for mathematics on the same lines as those being produced in Bexley. In many cases, the school has discarded its own scheme and replaced it by these guidelines or schemes by choice, but some Authorities have made it compulsory that their guidelines or schemes should provide the basis for mathematical work in schools.

Schools' need for Guidelines or Schemes

All the schools felt there was the need for some sort of scheme or guidelines to be available for the teacher's reference. It helped the teacher to see where he was going and what he should aim for; it gave him a feeling of security. It was also felt to be useful when there was a changeover of staff. The new teacher could fit in with the children, and he would know what had previously been covered. In the absence of an official scheme from the Borough, the schools tackled the problem in similar ways. The Head and staff got together to discuss their ideas. In some schools, one teacher has special responsibility for mathematics, and this may include producing a scheme for the rest of the staff to consider.

In other schools, the Head alone, or with a few knowledgeable members of the staff, produced a scheme. Once an approach had been decided upon and a scheme agreed, all teachers become more critical of their work and try to evaluate their progress in terms of achievement.

Most schools have staff meetings regularly — once a week or once a fortnight – and mathematical problems will be discussed at such meetings. Occasionally, there will be special meetings to discuss a particular aspect of the curriculum, and again mathematics is often chosen.

Advice about Schemes from the Borough's Advisers

When guidance was sought from the Borough's Advisers, it was suggested that the school examine the series "Mathematics for Schools", edited by Harold Fletcher (Appendix II), paying particular attention to the Teacher's Resource Book (this series will be referred to as "Fletcher"). Level I is for the Infant school and Level II for the Junior school. At the Infant level, the one Teacher's Resource Book covers the seven work books provided, in which the children can do their written work. At the Junior level, the ten children's books are text books. These are supported by five Teacher's Resource Books, one for each pair of children's text books.

At both levels, the material in the children's work book and text book is broken up into short sections, three or four pages long, each dealing with one idea. The Teacher's Resource Book examines each section in turn, giving teaching notes. Firstly, the purpose of each section is stated. This is followed by a list of preliminary activities which should be done before the child attempts the relevant pages in his work book or text book. Suggestions as to how to teach the pages are made. And follow-up activities, which further test whether the purpose has been achieved, are also mentioned. The Teacher's Resource Book further lists the objective of teaching the mathematical idea, the mathematics involved, teaching aids and materials which should be available for the activities, and gives the teacher further suggestions for general activities as well as for games, songs and some special activities. The time which could be spent on the section is also discussed, the point being made that sufficient time must be allowed for the children to gain a thorough understanding of the section. There are also enrichment activities for many of the concepts. "Fletcher" is an integrated series incorporating modern mathematical ideas and using these as a basis for the whole series. It has been written by practising teachers who have tried out the material in their schools. With the wide variety of experiences mentioned, it is possible for each child to progress at his best rate.

Schools using "Fletcher"

Seven of the schools are using "Fletcher" extensively but not exclusively — 3 Infant, 2 Junior Mixed and Infant and 2 Junior schools. They have chosen it as the basic book because they felt it is well-structured, has progressive stages, and offers very considerable support for the teacher. Schools were cautious in its introduction. In one Infant school, the teachers had met every Tuesday and discussed each stage in turn. The other Infant schools had introduced it gradually starting with the lower classes. Children entering both the Junior Schools had come from Infant schools which used it. One of the Junior Mixed and Infant schools tried it out first in the Infant Department but, in the other, it was introduced throughout the school at the same time.

Some Infant schools did not use the work books for the children's written work, preferring to supply their own sheets for this purpose. Several reasons were given — the considerable expense caused by using the books as disposable items was felt to be unjustified; the pages were of too formal a nature, particularly in Books 1 to 3; for the poor reader, even the limited amount of written instructions often presented difficulty. The series tried to cope with this by having a cartoon showing a child doing the same thing as the child was expected to do in completing the pages and also by having a symbol after several pages which indicated that the child should talk to the teacher at this point. Some Infant schools used the Teacher's Resource Book as a source book for the preparation of their own material, which they then incorporated in their own work cards or teaching. All Infant schools stressed that before the children attempted a page in the work book, some practical experience was essential. Teachers varied considerably in the amount they did, some believing too much was suggested.

As Level I was not published until 1970 and as Level II, Books 9, 10 and the accompanying Teacher's Resource Book will not be available until Spring 1975, it is too early to form a judgment as to how effective the books are.

Schools not using "Fletcher"

Of the four schools which were not using "Fletcher", one Junior school had chosen a series to use throughout the school "Making Sure of Mathematics" (Appendix II) before "Fletcher" had been published. This school was, however, now introducing "Fletcher" into its lower classes as money became available for its purchase, although the Infant school from which their children came did not use it. The second Junior school had rejected "Fletcher" in favour of the series "Basic Mathematics" (Appendix II). The Infant school had produced its own scheme, the Head and a teacher having worked together — no particular book was used but a variety of books was available in the staff library for reference, including "Fletcher". The Junior Mixed and Infant school had an outline scheme, currently being revised, available if anyone wanted to consult it, but staff had freedom to plan their own programmes as they wished. Several teachers might work together to ensure there was a progression. The Head in this school felt that this offered opportunities for the individual strengths of teachers to be used to the full.

Additional Material in Use

It was felt that, with a basic scheme in use, there was some danger that the children would just work through the pages of the work book or text book. To counteract this, all the teachers supplemented the text books by using other materials. One Infant school using "Fletcher" also used a set of commercially produced work cards "Outset" (see Appendix II), integrating the two schemes together and producing their own work sheets as well. All schools omitted sections of the series because they disagreed with the approach to a particular section or felt that it was unnecessary to the development they wished to follow — a section on "sequential patterns" in "Fletcher" was mentioned. Other sections were felt not to be developed enough, and these were reinforced with extra work from other sources. Sometimes, the order in which the subject matter was done was changed. It was felt important that the teacher should make suitable cards so that reading was minimised when there were children with reading and other difficulties.

Several schools had small sets of a variety of text books available from a central store so that children could cover the same topic from a second book but from a different angle. The teacher could also select more interesting or more suitable material on certain mathematical ideas. One school kept a selection of older type books finding them useful for the four rules — they gave more systematic practice than some of the more modern ones, which were described as "airy-fairy", "too piecemeal". It was found also that some pages were too overwhelming, containing so much reading matter that assimilation was impossible. Some books were chopped up and used as work cards.

Although there are a number of commercially produced sets of work cards on the market, these were not widely used. Two Junior schools had one set each but did not make regular use of them (see Appendix II).

Many teachers, however, used their own cards. They may be duplicated sheets, or they may be elegantly produced cards, illustrated and covered with varnish or takibak (transparent self-adhesive material) for protection and longer life. The work card might be for additional practice in a certain skill, e.g. more calculations involving addition. But more frequently it was for the teaching of items not included in the book being used. Teachers might also develop a set of cards for use with a particular piece of apparatus. An Infant teacher had produced a set of about 10 cards for use with the balance scales, graded very carefully in small steps. The first card showed the scales unbalanced, and invited the child to find objects which fitted the situation. The last card asked the child to find out how many cubes were needed to balance the parcel and then write down a statement describing the situation " cubes balance the red parcel". A Junior teacher used an equaliser with children who had difficulty with the basic number facts. The first card showed an illustration on the equaliser of the sum "5 = 2 + 3" and invited the child to test the truth of the statement. Later cards asked the child to find the missing number required to make the equaliser balance, "4 + □ = 6", "8 = 3 x □ + 2".

Topic Work

In addition to work from the basic series or work cards, many mathematical opportunities arose from project work. Some classes concentrate on a project each term, e.g. transport, choosing the project so that as many aspects of the curriculum as possible can be integrated into it. Mathematics featured quite prominently in many of these topics. The teacher might isolate some mathematical ideas beforehand when considering what aspects to concentrate on. She would then prepare work sheets or cards which would direct the children's attention to these particular situations. From the topic of "Transport", mathematics arose in many ways:

- measurement of length and area (from a consideration of the size of different modes of transport);
- the speed of vehicles;

- geometrical shapes, particularly the circle, arising from an initial consideration of the wheel;
- scale models of various vehicles;
- graphical representation of a survey of traffic going past the school.

A visit to a windmill by a class of Infants involved mathematics on the way there and back, as well as the mathematics of the windmill itself. The children made some counts — the number of post boxes, telephone kiosks, and vehicles on the road they passed, the number of steps within the windmill. They estimated the height of the windmill, and measured the air pressures on the sails and the speed with which they turned. They also estimated the grain capacity, and worked out how much the grain would weigh.

On a visit to a local castle, plans and charts were made of the site and work was done on maps. This included a discussion of scales.

In a social studies topic, centred on the family and the home, much statistical work emerged, leading to extensive exploration of methods of graphical representation.

When one class was preparing for Christmas, and the question of decorations arose, the children decided to make geometrical models for display and as packages for presents. They also made stained glass windows using ideas of axial and rotational symmetry.

Children who are used to thinking mathematically for themselves often discovered more mathematics in the topic than the teacher had originally intended. This was felt to be one of the valuable results which had come from the new attitude towards mathematics. Children were felt to have a much more positive attitude and a greater readiness to tackle incidental problems not previously explored and often involving new ways of thinking and learning.

Television Mathematics Programmes

A number of Junior schools make use of television mathematics programmes. One school takes ITV's "Figure it out" for 7 to 9 year olds, which is shown weekly throughout the school year. Several schools used the BBC's "Mathematics Workshop", stage 1 (for 9 to 10 year olds) and stage 2 (for 10 to 11 year olds) which is shown fortnightly. The programmes lasted about twenty minutes. The teachers' booklets provided contain a summary of the ideas presented in the TV programme, with a booklist for further reading (for both teacher and child) and suggestions for follow-up work which can be developed from these ideas. Each programme was independent of the others so that teachers need not take all of them. For the BBC series, work cards were available. These contained work directly linked to the TV programme.

Other teachers would have liked to use the TV programmes but were unable to do so because the TV set or perhaps the room where it was kept was unavailable (no school had a video recorder).

One teacher uses the work cards, even though the children have not watched the TV programmes. The cards do not mention the TV programmes so it is possible, after some discussion with the teacher about the subject matter, to use them. If the teacher uses all the suggestions made for follow-up work, there is enough work to keep the children busy between programmes, without doing any other mathematics.

Books for Reference for Teachers and Children

Teachers were appreciative of the help and guidance as well as the multitude of suggestions available in the Teacher's Resource Book provided with "Fletcher". They were critical of those teachers' books provided with other series which were usually little more than answer books. (Even these were often incorrect.)

In all the schools, there were libraries — both in the classroom for class use and in the school for general use. Most contain books about mathematics for the children (some are listed in Appendix II). There are also staff reference libraries. These contain not only one or two copies of children's text and topic books, but books about teaching mathematics for teachers and books containing suitable content for the Infant or Junior school, e.g. the Nuffield guides (Appendix II).

Unlike a text book which contains information and questions on a variety of mathematical ideas, in a "topic" book all the material is concerned with one idea, e.g. "Cubes" from "Topics from Mathematics" (Appendix II).

The Teacher's Role in the Classroom, assessing and marking work

All teachers felt that it was important to keep a constant check on the work both written and practical, that each child was doing. The teacher moved round the class (or the child might go up to the teacher's desk), checking what an individual child was doing (even when he was a member of a group). Incorrect working was pointed out, and had to be re-checked. Usually the child repeated the activity as a further check. A wrong or careless use of a piece of apparatus was discussed, and the child watched to see that he was now using it correctly. For a child in difficulties, not understanding a question or being unable to complete a process, the teacher would ask supplementary questions, give an extra hint or ask a question about previous work to make sure the child had understood.

One Head commented that he preferred the *process* to be marked rather than just the answer. A second Head said he thought work should be "remarked" on, rather than just marked. The teachers tried to mark most of the children's work with the child present so that any difficulties could be discussed directly, whilst it was fresh in the child's mind. They felt that this was more helpful to both the child and the teacher, particularly when the child was young. Even so, in some cases, correct answers were accepted as a sign that the child had understood the process without him being questioned on it. In many cases, the teacher felt that his main role was to guide the child through the sets of examples provided in the book and that, if the child did these successfully, he would understand the mathematical processes involved and be able to use them satisfactorily in other situations. Others took a wider view, and felt that, while it was vital that the child should acquire skills, the understanding of mathematics processes required discussion and thought, as well as involvement in practical work on the part of the child.

Where there was a definite answer to a piece of work, in many classes the children were encouraged to check this for themselves. The teachers felt that this saved time and meant that children developed a feeling of responsibility. Other teachers felt that children would cheat, and simply copy the answer without working out the problem.

Records

London Borough of Bexley Official Record Card

The London Borough of Bexley has an official record card, which is completed by the Head when the child leaves the Infant school and again when he leaves the Junior school at the age of 11. This record goes with the child to the new school. There are two sections for mathematics — one for experiences and one for recording.

Schools Official Records

Three schools also kept an official school record. One Junior school had a section labelled "Mathematics"; it was completed as the child moved through the school. A second Junior Mixed and Infant school had sections for aspects of mathematics, viz. sets, number (place value and four rules), length, perimeter, weight, capacity, money, time, shape and space, area, volume, symmetry, rotation, graphical representation, vulgar and decimal fractions: percentages, with reference to understanding and accuracy throughout (Appendix III). This record followed on from the one kept in the Infant school where vocabulary and basic concept were noted, with one tick being used if the vocabulary and basic concept had been met, and two ticks if they had been understood. In a third school (Infant) record files were kept, starting when a child entered school. Teachers were asked to put down what they were fairly certain the child knew — they wrote in the file as things happened. The Head kept the files constantly under review, and revised them from time to time with the staff.

Teachers' Personal Records

Several schools did not feel an official record, other than the one required by the Borough, was necessary, but all Heads expected teachers to keep a personal record of some sort. It might simply be a note of the work cards the child has completed, the position he has reached in the series of books the school is using, a list of experiences he has had. Or it might be a list of what the child has not understood or has missed through absence. One Infant school said "Fletcher" in itself was a record — each term a sample of work is put into the child's progress folder, plus a list of other experiences and details of "Fletcher". In a Junior school, the teacher was required to forecast what work he expected to cover in the next half term, and submit this plan to the Head and discuss it with him. At the end of the half term, the teacher then discussed what had been achieved with the Head. The discussions were very informal.

When a teacher is leaving or a child is changing schools, a full written record is required. Otherwise, if the child is changing classes, the teachers can talk to each other informally. Concern was expressed that the records kept were not adequate. It was easy to record what experiences the child had met and what problems he had done, but, if a child had worked out and recorded the fact that $4 + 5 = 9$, what did this mean? It was difficult to evaluate what the child had understood. No-one had a completely satisfactory answer, even though discussions about record keeping were fairly continuous.

Testing Children

There is a verbal reasoning test but no mathematics test in the Borough for children leaving the Primary school. Within each school, and subsequently within the Borough, the children are put in an order of merit. At present, the Borough has selective schools as well as comprehensive schools and a certain percentage of children at the top of the scale of merit enter these selective schools.

Several schools administered an NFER test (Appendix II) to help them make a decision, others used their own test, yet others looked at the work the children had done and made their judgment on that. One school devising its own test, based it directly on the work the children had done. Another school felt that, in order to be able to do a commercially produced test, the work would have had to be done in strict sequence and this method of working they rejected (they were one of the schools which did not use "Fletcher"), so they also rejected the test! In lower classes in the Junior school, there was no testing done of a formal nature. In the Infant school, there was some testing done on the lines of the Nuffield Check-Ups (Appendix II), sometimes by the class teacher, sometimes by the Head who felt he was more independent. One or two schools felt there was no time to do testing.

IV. THE DEVELOPMENT FROM INTUITIVE TO MORE FORMALISED WORK

Since there is no mathematics test given by the Borough to children when they leave the Primary school at the age of 11+, teachers are no longer required to teach certain mathematical ideas. There is much more freedom and flexibility to teach what one wishes. Some Secondary schools, having their intake of 11 year olds from many different Junior schools, find this creates a problem, and would wish that there was more uniformity in the content of mathematics as well as in the methods of teaching mathematics in the Primary schools. However, these comments made by Heads show that schools are very much aware of the need for progression within mathematics, particularly in number work:

- "Mathematics is very logical and should be taught logically";
- "There is need of a framework to draw all topics in";
- "If children can understand the numbers 1 to 20, the Infant school has done well in laying down a foundation";
- "By the end of the second year Juniors (about 8 or 9 year olds) should have a good working knowledge of the numbers to 200";
- "We have got to keep going back to reinforce what the children have done."

The Philosophical Approach to Mathematical Teaching in the Primary School

Insufficient opportunity was available to explore the teachers' personal attitudes and views on the general principles of mathematics teaching. Nevertheless, all the evidence suggests that certain broad lines of development would be commonly subscribed to and generally accepted. The evidence lies in the choice of books and apparatus and in the day-to-day activities in the classroom rather than in any verbal statements made by teachers. A study of the books used for basis schemes (listed in Appendix II) will give details of the mathematics schools aim to cover. Even so, it seems worthwhile to attempt to set out some of these general principles, which appear to constitute the basis for much of the methodology of mathematics teaching in English Primary schools.

In order to learn mathematics effectively, children must be presented with situations involving concrete materials from their environment which are real to them. They may need to be helped to read mathematical material.

They must be encouraged to verbalise their ideas as well as they are able. Until children can express orally what they are doing, they are not ready to practise "sums", etc. If this order is followed, then, when symbols are introduced, they can take on immediate meaning. The standard written notation may be introduced gradually, e.g. starting with "2(4) → 8", then "2 x 4 → 8", leads finally to "2 x 4 = 8".

It is important that the child should have a thorough understanding of concepts, and then be able to apply these concepts to new situations. After the discovery activities, there must be plenty of practice and time to develop and consolidate understanding of mathematical concepts. Because of the nature of mathematics, subsequent understanding is based on how well the previous concept has been understood. For example, acceptance and understanding of the concept of conservation of number by the child indicates his readiness to begin the study of the cardinal aspect of number, in an intuitive discovery situation. Understanding of this will, in turn, lead to the exploration of addition in a discovery situation. It is little use expecting a child to understand multiplication, however, if he has not understood addition. Neither is it any use a child knowing the multiplication facts, i.e. "tables", if he does not know how or when to use them. If he sees the multiplication sign, he may be perfectly happy. But, if he has to translate the problem into symbolic form and cannot do this, then he is not able to use what he knows.

The introduction of metrication has caused all schools to rethink their work on the measures. The measures are more directly related to the home and the environment than any other part of the mathematics curriculum. Because the metric unit is often smaller than the equivalent Imperial Unit (compare the length of 1 cm with 1 inch), many Infant schools have delayed the introduction of the standard unit until the child is about to enter the Junior school, with a consequent delay in the need to handle larger numbers. However, it will provide earlier experience (in the Junior school) with decimals and calculations involving them. Some teachers have yet to be convinced that this is useful. Many teachers describe the "•" in £3.24 as being the division between pounds and pence without discussing its proper significance, thereby missing one of the "real-life" applications of decimals.

Many teachers favour common methods of approach to provide a certain uniformity in processes. Subtraction by "decomposition" is thought to be the method most naturally following the use of environmental material. Teachers felt, too, that as many children as possible should be using "standard" methods of long multiplication and long division by the time they left the Primary school. Whether this kind of uniformity is desirable is open to question. A number of children struggle continually with their work on numbers and do not achieve competence in the four rules of number before they leave the Junior school. Teachers felt that, whatever level the child reached, what little he had accomplished should be as thoroughly understood as possible, but this did not prevent them from introducing processes and the child practising them before he had fully understood.

It is important, too, that children should have experience of some open-ended situations where several correct answers are possible. This stimulates discussion and encourages children not to assume the problem is solved when one solution is found. This further relates to real life, where frequently there are alternatives to choose from and not all the data is relevant or needed.

V. THE PLACE OF ENVIRONMENTAL AND STRUCTURAL APPARATUS

When a school had decided what mathematics scheme it would follow, particular attention was paid to the apparatus and materials which would be needed for it. All schemes and guidelines stress the need for a variety of apparatus and material for each process. Suggestions for teaching aids and materials for each section are given in the Teacher's Resource Book of "Fletcher". In other cases, the equipment needed to work through the book is listed. If this is not given, the teacher with the special responsibility for mathematics will go through the books and make a list of all the equipment that she thinks will be needed.

Much material can be used in a variety of ways for different mathematical ideas, but some apparatus is likely to be used only for a particular idea — a trundle wheel would be unlikely to be used for anything other than measuring a distance along a path or road. One Junior school lists the mathematical equipment available in the school under six headings — weight, time, measuring (meaning length), money, capacity and general. Included under the general heading are pieces of apparatus which would be used in conjunction with that listed in the other sections and for work on number and shape. In fact, one of the avowed aims of the schools in deciding what materials shall be bought is that they shall be as flexible as possible in their use. There is no guarantee, however, that a piece of apparatus once available in the school will be used. Even structural apparatus such as Cuisenaire rods is used for a variety of purposes not directly concerned with number.

It is not the materials themselves which are important but the use which is made of them. It seems to be generally appreciated that it is desirable, and perhaps even essential, that each mathematical idea be illustrated in several ways.

ENVIRONMENTAL APPARATUS

Environmental apparatus plays a very important part in mathematics because it can bridge the gap between the real and familiar world and the structured situation when the child begins to formalise concepts and to formulate valid generalisations. The new methods for teaching mathematics could not have been developed so successfully without environmental apparatus. Teachers can make some for themselves, but not everything can be constructed easily and accurately. The materials used for play, mentioned in Section II, can be described as environmental apparatus. The wide range of materials available for use in the schools for measuring emphasises the importance this aspect of mathematics holds.

To gain a full understanding of the nature of the measures, the child must have experience of small quantities as well as larger ones. He must have sufficient practical experience to be able to imagine quantities so large or small although he cannot actually see them in reality or measure them.

Weight

In consequence, different types of scales and balances are used to cater for this range. There are letter scales sensitive to smaller weights, graduated in 10g, and personal scales, graduated in 500g. Between these two extremes, there is a variety of balances, compression and extension scales and sets of weights to use with them. The child learns to select the type of scales best suited for his purpose, and the weights, if necessary, to use with the scales.

Length

There are perhaps more different pieces of apparatus concerned with measuring length than with most other measures. Metre tapes and sticks, surveyor's tapes up to 33 metres in length, and trundle wheels are familiar pieces of apparatus, found in all schools and most classrooms. There are also height and foot measurers which make it easier to take these measurements of the body. Large wooden calipers and small metal ones measured diameters and widths of large objects and the thickness of a hair or a piece of paper.

Capacity

So many products are sold in containers which are marked with the capacity of the container that it is not so necessary to buy specific apparatus for this measure. Much material can be calibrated to a reasonable degree of accuracy by the children themselves. However, standard plastic measures range in size from 1 ml to 2000 ml. There are many sets which have selected sizes, e.g. 1 litre, 500 ml, 250 ml, 100 ml, 10 ml, stacking inside each other.

Time

Clock faces with adjustable hands for indicating a time are used in the Infant school. Clock face stamps are available so that children can record a time without the bother of drawing a clock face each time. Various instruments for measuring time were found — seconds timer, stop-clock, stop-watch, sand timer, tocker timers (5, 10, 15, 20 seconds). The school day is no longer ruled by time in the way that it used to be but nevertheless time plays an important part in life. Most classrooms had a working clock.

Money

The handling of coins of the realm is recommended as a better motivation for learning about money, but this is not always possible or desirable. Both plastic and cardboard coins look reasonably like the "real thing" and can be available in larger quantities than it would be possible to have of real money. Often a cash register will be available for use in the shop.

Shape

Protractors, set squares, pairs of compasses, are necessary tools for construction work. Card and gummed cut-out geometrical shapes, as well as plastic ones, provide immediate visual pictures for making patterns and constructing three-dimensional solids. Collections of manufactured wooden solids are sometimes seen but empty junk containers can provide the majority of materials needed for this. Geo-strips (strips of card or plastic) with holes punched at certain positions are used to construct two-dimensional frameworks to illustrate, say, the rigidity of polygons. Meccano strips can also be used for this work, but the holes are equally spaced throughout the length.

Number

There are no items of environmental apparatus used specifically for number, but much of the general apparatus will be used.

General

There are several pieces of apparatus, which are so versatile that they can be used for a wide variety of purposes.

Wooden and plastic cubes can be used for counting, making patterns in two and three dimensions, for building, for filling containers, as dice, for covering surfaces and to measure areas.

Pegboard can be bought at "do-it-yourself" shops and cut into the required size, but 10 x 10 pegboards can be purchased in plastic (with sets of pegs). Uses include making patterns of 2's, 3's, etc., making geometrical patterns, illustrating block graphs, as a number line in contrast to a strip of card where numbers are represented by lengths.

Nailboards are available in many different sizes, mostly home-made and made from thick chipboard or plywood. The nails can be arranged in any formation, but square formation is the most common. The smallest size is 9 nails in a 3 x 3 formation — the nails are spaced up to 5 cm apart. The largest one which can be profitably used is 20 x 20, the nails being spaced about 2 cm apart for ease of handling. Shapes are enclosed by rubber bands (preferably coloured). Area of the shapes can be found. Different types of polygons can be isolated and then counted on boards of different sizes. The board can be used to illustrate concepts of symmetry and for finding which shapes fit together and which shapes can be made on the board. Useful concepts of size, shape, congruence and similarity can all be developed, as well as much practical manipulation with fractions.

Summary

Many more pieces of apparatus could be described, but it is hoped that the items mentioned will serve to illustrate the wide range of materials which are in use and which contribute not a little to the child's understanding of mathematics.

STRUCTURAL APPARATUS

The most common piece of structural apparatus found in the schools was the *number line*. It appeared in many positions around the school as well as in different sizes — small individual ones on the children's desks, larger ones round the wall of the classroom, and larger ones still, drawn out in the playground. None was numbered beyond 100. For the most part, these were made in the school, either by the teacher or the children. These lines had many uses. One was as a reference for the child to check that he had written a particular number correctly, or to check the result of a calculation, using the relevant strips. Or it might be used to aid a child in performing a calculation by providing a pattern of numbers and enabling him to count either backwards or forwards. Counting games are played on the number line in the playground. It is regarded as an essential piece of equipment in the development of the child's understanding of number.

Apart from the number lines and the Dienes logical blocks, all other structural apparatus noticed in the schools has been specifically manufactured as an aid to number work. Manuals of instruction are provided but schools did not use the material in the way these suggested. In many cases it was used simply as counting or sorting apparatus.

Unifix apparatus was found to be the most common and most widely used.

This apparatus was regarded principally as apparatus for the Infant school but many Junior schools also used it. Apart from the interlocking plastic cubes, in ten colours (the cubes are about 2 cm in size), which can be joined to form lengths, a wide range of accessories is available. The cubes can be bought by the hundred without the accessories. These seemed to be little used. Unifix cubes are used as units of measures, in sorting activities, for making block graphs and patterns, as well as for number work. The *counting* aspect of number is emphasised. If the child is working out "8 + 5", he counts out a set of 8 cubes, then a set of 5 cubes, puts the two sets together and counts the total. Being able to choose any mixture of colours, it may not occur to the child to build up his rod of 8 cubes in one colour and his rod of 5 cubes in a different one. By the time the child comes to use the cubes for making "tens and units", he will have realised the usefulness of doing just this.

Stern blocks were seen in one or two schools. They are wooden (with a cross-section the same size as the Unifix cubes), in ten different lengths, each length a particular colour, marked so that the number of cubes in each block can be counted. There are many different coloured cubes in the set. Sets contain a specific number of blocks. This apparatus also has many accessories, again not seen in use. To work out the problem "8 + 5", the two appropriate rods are selected, laid end to end, and measured against a "10" rod. The missing rod is then selected to complete the matching.

A set of *Cuisenaire* rods, made in 10 different lengths, all different colours, 1 cm cross-section, contains over 200 pieces. There are no accessories. They are unmarked along their length. Children learn to recognise a red rod as being of a certain length. They are used in much the same way as Stern rods. In some cases, they were used just as counters. However, they were thought to be a useful aid in measuring, as their lengths varied from 1 cm to 10 cm, in 1 cm changes.

One hand-operated calculating machine was very popular. Children used it to perform calculations involving large numbers which they checked by working out the sum. The teacher was impressed with the children's industry! These machines are still an expensive item of equipment, so it is unlikely they will be widely used in Primary schools.

The equaliser is a useful piece of apparatus if it is reasonably accurate. It can be self-checking, and it can help the child to manipulate numbers on each side of the equation.

Several schools had sets of *Dienes Multibase arithmetic blocks* (Dienes M.A.B.) or *Tillich blocks.* The material for base 10 was thought to be the most useful, although that for other bases was also used. Teachers felt that, in the Junior school, this type of material reinforced or extended the ideas of place value the child gained from using the other materials. It was also felt to be a step towards the abacus, its main virtue being that 10 "unit" pieces could be seen as 1 "ten" piece and could replace it, the same amount of material being used. And similarly, 10 "ten" pieces could replace 1 "hundred" piece and 10 "hundred" pieces could replace 1 "thousand" piece.

Several abaci were noticed — both hoop and spike form — some home-made. The children use them by positioning the beads to show various numbers and performing simple calculations. Illustrating the number "146", only 11 beads are used, providing a valuable contrast to the Dienes M.A.B.

Dienes logiblocs are used to give children sorting experience of various types. Unlike the environmental material which is also used, the ways in which the logiblocs can be sorted are limited but the differences (up to four) between the pieces, in various combinations provide extensive experience. They are regarded as a useful piece of apparatus at several levels.

Summary

It was difficult to judge whether the use of these pieces of structural apparatus contributed as much as they might have done to number work in the Primary school, since they were used so unsystematically.

APPENDIX I

LIST OF SCHOOLS VISITED

Bedonwell Infant; Headmistress – Miss M. C. Surridge

Danson Junior Mixed and Infant; Headmaster – Mr. E. G. Firman

East Wickham Junior Mixed; Headmaster – Mr. D. F. K. Champ

East Wickham Infant; Headmistress – Mrs. H. M. C. Ponting, M. A.

Gravel Hill Junior Mixed and Infant; Headmistress – Mrs. E. M. J. McGovern

Hurst Junior Mixed; Headmaster – Mr. E. K. Cartwright

Hurst Infant; Headmistress – Mrs. M. M. Scrogie

Mayplace Junior Mixed and Infant; Headmaster – Mr. T. J. Gibbs

North Cray Junior Mixed; Headmaster – Mr. R. P. G. May

North Cray Infant; Headmistress – Miss R. Davis

Royal Park Junior Mixed; Headmaster – Mr. C. H. Lambert.

APPENDIX II

A SELECTION OF BOOKS NOTED IN THE SCHOOLS

Books marked * were used for basic schemes.

Association of Teachers of Mathematics:
Notes on Mathematics in Primary Schools: C.U.P.: 1967.

Brayton, Howard:
Topics in Practical Mathematics: James Brodie: 1968.

Clark, Mollie:
I can count.
I can measure.
I can weigh: Wheaton: 1958–1960.

* Cormack, C. D. and Fraser, I. F.:
Making Sure of Mathematics Introductory and Workbooks for Infants (4 books): Holmes McDougall: 1970.

* Epps, P. and Deans, J.:
Mathematical Games: Macmillan: 1973.

Flavell, J. S. and Wakelam, B. B.:
The New Primary Mathematics: Methuen: 1969.

* Fletcher, H. ed.:
Mathematics for Schools,
Level I Books 1–7 and Teacher's Resource Book,
Level II Books 1–10 and Teacher's Resource Books:
Addison-Wesley: 1970 onwards.

Foster, Leslie:
MacDonald Starters Mathematics (various titles):
MacDonald Educational: 1973/74.

French, P. and Rickard, R. J.:
Exploring Mathematics Series (12 books):
McGraw Hill: 1966.

* Gloucester Education Committee Mathematics Cards Scheme:
"Outset" Work Cards: Macmillan: 1973.

Goddard, T. R., Grattidge, A. W. and others:
Alpha Mathematics Books 1–4,
Beta Mathematics Books 1–6:
Schofield and Sims: 1971/72.

* Griffiths, A. L.:
Basic Mathematics Books 1–4: Oliver & Boyd: 1970.

Hesse, K. A.:
The four rules of decimal money: Longmans: 1969.

Marsh, L. G.:
Let's Discover Mathematics Books 1–5 and workshop cards 1, 2, 3: A. & C. Black: 1971.

Mold, J. and Fielker, D. S.:
Topics from Mathematics (various booklets): C.U.P.: 1967 onwards.

National Foundations for Educational Research in England and Wales:
Mathematics Tests: Ginn.

Nuffield Mathematics Project:
Introductory Guides (3 vols.),
Main Teachers' Guides (12 vols.),
Weaving Guides (6 vols.),
Check-Up Guides (3 vols.),
Parents' Guide (1 vol.),
Problems (3 sets of cards): Chambers/Murray: 1965 onwards.

Razzell, A. G. and Watts, K. G. O.:
Mathematical Topics (Books 1–6): Hart-Davis: 1964.

Saunders, J. G.:
Mathematics Topic Cards: Nelson: 1970.

Sealey, L.:
Let's think about Mathematics Books 1–4: Nelson: 1971.

Sealey, L.:
Let's find out about Mathematics Books 1 & 2: Nelson 1973.

Schools Council:
Mathematics in Primary Schools: H.M.S.O.: 1965.

Taylor, J. and Inglesby, T.:
Number Words: Longmans: 1963.

* Watson, T. F. and Quinn, T. A.:
Making Sure of Mathematics (6 vols): Holmes McDougall: 1967.

Williams, E. M. and James, E. J.:
New Oxford Junior Mathematics Books 0, 1, 2, 3, 4, 5: C.U.P.: 1971.

Various authors:
Ladybird handbooks (series of titles): Wills & Hepworth.

APPENDIX III

NUMBER WORK (with reference to understanding and accuracy)

SETS

NUMBER (Place value and four rules)

LENGTH

PERIMETER

WEIGHT

CAPACITY

MONEY

TIME

SHAPES AND SPACE

AREA

VOLUME

SYMMETRY

ROTATION

GRAPHICAL REPRESENTATION

VULGAR AND DECIMAL FRACTIONS: PERCENTAGES

A SUMMARY OF THE OBJECT AND SCOPE OF THE CASE-STUDY, THE LINE OF REASONING ON WHICH IT HAS BEEN BASED AS WELL AS THE MAIN LESSONS AND CONCLUSIONS TO BE DRAWN FROM IT

The study has been restricted to one Borough in England, namely the London Borough of Bexley. This Borough has 74 Primary schools – 32 Junior Mixed and Infants, 24 Infants and 18 Junior Mixed. In the Infant school, children's ages range from 4+ to 7+, and in the Junior school from 7+ to 11+. (The Junior Mixed and Infant school contains children from 4+ to 11+.)

It was felt preferable to make a study in depth of a limited number of schools rather than a superfical study of a great many. In consequence, 11 schools were selected from this total with the help of both the Mathematics and the Primary Adviser from the Borough — all 11 schools were known to have a particular interest in Mathematics. (This is not to say that they do not also have particular interests in other parts of the curriculum or that there are not other schools within the Borough which also have a particular interest in Mathematics.) The schools selected consisted of 3 Junior Mixed and Infant, 4 Infant and 4 Junior schools.

During the visits, there were discussions with both the Head and the teachers. Classrooms were visited and children observed at work. There was a change of staff in some schools over the summer vacation, but the Head of each school remained the same. Of the 11 Heads, 1 Junior Mixed and Infant and 4 Infant were women. There were considerably more women than men teachers. Two Heads and several teachers were already known to the interviewer. The purpose of the initial meeting at the school with the Head was to find out about:

i. the organisation of the school;

ii. the Head's attitude towards mathematics;

iii. the nature of the mathematical scheme in operation;

iv. the kind of methods of teaching mathematics encouraged;

v. the way in which methods were used;

vi. the extent of teacher participation in decision-making in relation to mathematics;

vii. the relations the school had with other schools and with parents;

viii. the amount of apparatus and materials for mathematics in the school.

As continuous observation in any school could not be maintained, the picture of the mathematics being done is necessarily fragmented. In visiting the classrooms, efforts were made to confirm, by direct observation, the views already expressed by the Heads in conversation and more practical details were checked directly, such as the physical arrangements of the room and how far these facilitated mathematics learning. Children's attitudes were also observed and the handling of mathematical ideas by staff and pupils.

The teachers were very willing to discuss their attitudes to mathematics and the teaching of it, and the children to talk about the on-going work they were doing. There is a considerable amount of diversity in the use of materials and methods in a school as well as in different schools. Many of the schemes had only been operating for a relatively short time, and no-one felt that they could pass an opinion on their success or failure until several more years had elapsed. Teachers found that the idea of using "Fletcher" was overwhelming at the beginning, but many remarked that they were getting more used to it as time elapsed.

A system which allows individual schools and individual teachers to follow their own ideas is possibly as weak as its weakest teacher. All Heads felt that it was their responsibility primarily to see that every teacher had as much support as she needed. It is likely that in the present climate, where specialist knowledge of the teaching of mathematics in the Primary school is rare, schools will more and more try to structure their mathematics teaching and will rely on guidelines and schemes to give their teachers support. On the other hand, the present freedom allows teachers with considerable strengths to use these strengths to the full. There would seem to be considerable hope for the teaching of mathematics where there are teachers who think and plan so carefully as those in the eleven schools studied.

The main conclusions drawn from this intensive but localised study of mathematics teaching in Primary schools are as follows:

i. *Use of experience and play for young children*

Learning mathematics in the Infant school was largely, if not wholly, through play activities, based on a combination of teacher constructed and designed and commercially produced games and apparatus. Such activities steadily diminished throughout the Junior school, although projects in which mathematics played a central part were fairly common.

ii. *Individualised and group learning*

Throughout the Infant schools, the classes were organised entirely in terms of individual and group activities. Many Junior schools had adopted some form of "integrated day". Mathematical activities fitted into the general pattern of school or class organisation, except in one case where the children were "setted" in ability groups for mathematics. At the upper end of the Junior school, there was more class teaching but also a great many class activities to which organised groups of children contributed.

iii. *The development from intuitive to more formalised work*

This has been provided for, in the main, by

1. the choice of a well-structured modern series of text books covering the age range 5-11, and supplemented by a comprehensive Teacher's Resource Book;
2. the drafting of "Guidelines" by a teacher organised group in the Borough (not yet complete);
3. individual school schemes and record cards.

iv. *The place of environmental and structural apparatus*

A wide range of measuring apparatus and measuring activities (related to length, weight, capacity and volume, time and money), were in fairly continuous use throughout the Primary school. Such activities have been reinforced to support the metrication of the measures. Extensive use was made of environmental activities to develop concepts of number. Commercially produced structural apparatus was usually available but in no case was it used in a formal or systematic way. A range of school constructed apparatus was, on the whole, found by the teacher to be rather more useful.

APPENDIX II

A CASE STUDY ON THE TRAINING AND RE-TRAINING OF TEACHERS OF NEW MATHEMATICS AT PRIMARY LEVEL

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APPENDIX II

Prefatory Note

Since the material available to the author was not entirely without gaps, the details which follow – particularly those relating to in-service training arrangements in the various *Länder* – are necessarily incomplete. In most cases, however, the general issues involved are unaffected by this circumstance.

Nor should the extent of the discussion in the present report lead to any conclusion as to the extent of such arrangements. The comprehensive treatment devoted to the position in *Land* North Rhine-Westphalia is intended to serve only as an example of further-training schemes and the amount of planning they require, and of the difficulties encountered at both the planning and execution stages.

The 1968 Recommendations of the Education Ministers' Conference (KMK) and the reform of mathematics teaching in the Federal Republic of Germany

Broadly speaking, the reform of mathematics teaching at all school levels in the Federal Republic, including the primary schools *(Grundschulen)*, has been persistently influenced, introduced and guided by the "Recommendations, guidelines and outline plans for modernizing mathematics teaching in general education establishments" (¹) (hereinafter referred to as the "Recommendations") adopted in a Resolution of the Standing Conference of Ministers of Education (KMK) at their 125th plenary meeting on 3 October 1968.

The "Recommendations" constituted the first major attempt in the Federal Republic to draft a mathematics syllabus for pupils in all categories of school from *Grundschule* to *Abitur* and to arrange the subjects covered in accordance with structural criteria. This was also the first attempt to co-ordinate the efforts of the various *Länder* to modernize mathematics teaching and to achieve a minimum of agreement. In the primary schools instruction in all the *Länder* on the lines laid down was due to become mandatory at latest by the start of the 1972-73 academic year.

Teachers and educationists held detailed discussions of the "Recommendations" and their implications at the Federal Conference on mathematics teaching (Ludwigsburg, 25-29 March 1969). It was especially emphasised that permanent contacts must be established and information exchanged between all committees working in the individual *Länder* on new guidelines for mathematics teaching in different types of school, and that adequate and comprehensive further training must be dispensed to the teaching body, the vast majority of whose members, it was estimated, were incapable of implementing the required reform, from the standpoint of either technical knowledge or teaching method.

Content of the reform

The concluding discussions led to findings which are important in the light of current public debate in the Federal Republic on the whole problem of modernising the teaching of mathematics, with the implied problems it poses for the in-service training of teachers. It was noted that mathematics teaching is today undergoing a phase of change and of adaptation to constant developments in the mathematical sciences themselves: old "subjects" must be replaced by new, and those that have proved their worth must be presented in another form and by the use of modern methods. Any such period of change is, of course, certain to be marked by differences of opinion.

Elementary set theory, in modern mathematics, holds a key position that is all its own, in that in all other mathematical disciplines concepts of the elementary theory of sets are used in order to provide clear and simple definitions of facts. One may even go so far as to say that modern mathematics is written in the language of the theory of sets. Hence on the one hand a certain minimum of concepts taken from elementary set theory is essential to the understanding of other disciplines, and on the other hand such a minimum, as determined by concrete use of the "set theory language", is sufficient.

Learning elementary notions of sets from practical experience, i. e. without the exposition of basic theory or indeed any theory at all by the teacher, is something which can and must take place in the school – whether *Grundschule* or *Hauptschule* (secondary school) – if it is to lead to a proper picture of modern mathematics. In reaching this conclusion, the Conference was not thinking of the set theory as a subject in its own right, but rather as an integral part of the syllabus, consisting essentially of "terminology" and "trivialities" and applied to other forms of subject matter. The aim, therefore, is to inculcate in the pupil a greater degree of clarity, simplicity and accuracy with the aid of elementary concepts of the theory of sets.

During the final discussion the following arguments were put forward:

1. Mathematics teaching in primary and secondary schools, as elsewhere, must start from the present-day picture of mathematics, in particular modern "elementary mathematics". This

implies the following requirements: integrated set theory, and the set theory as a "language" and means of expression – a certain minimum, but no more!

2. The elementary concepts of the theory of sets can be of some help in driving home, inter alia, the concept of number. If priority is given to cardinals, the basic notions of the theory of sets take on special importance. The other possibilities – ordinal and operative aspects – must not, however, be overlooked.
3. The elementary theory of sets is in a certain sense a "model" for logical demonstration. Some aspects of quantitative calculation may be useful in the treatment of problems of logic.
4. With particular reference to mathematics teaching based on modern methods, whereby the teacher does not regard himself so much as a purveyor of knowledge and understanding, but tends rather to become an initiator and stimulator of interesting, challenging and creative situations, the elementary theory of sets opens up promising avenues.
5. *Ad hoc* modernization, i. e. the purely formal and isolated introduction of "modern" concepts and methods into the teaching course, is to be discouraged. Only through reasoned and lasting use will concepts and methods, including "modern" notions of mathematics, be properly introduced into the classroom.
6. Accordingly no teacher should introduce into his classes modern concepts, or such as are new to him, until he has mastered their significance, applicability and advantages for his future lessons and can turn them to practical use. Herein lies the rationale of in-service training for teachers.

Criticism of the KMK Recommendations: their value and the conditions governing their implementation

On first analysis (²) the "Recommendations" were seen to betray a not inconsiderable bias in their reasons for advocating the modernization of mathematics teaching (inter alia: the ensuring of economic growth; shortage of teachers; the output demands of society; progress in mathematics). The critics also warned against the tendency to enthuse too much about the need to modernize the teaching of mathematics, a tendency that is most noticeable in the case of the primary school. In their own words: "One cannot exclude at least the danger of a one-sided approach, even though most educationists are certainly agreed that the theory of sets must be fostered as part and parcel of the system – not as an end in itself, but rather as a "language" of quantities: elementary notions of sets should be introduced inasmuch as they offer clearer and more accurate means of expression. There would be no harm done if the "Recommendations" came out more clearly in that direction; perhaps the first group (grades 1–6) could be treated separately in the syllabus and converted into a general team of pioneers. In order to counter the risk of further one-sidedness, it would also be useful to indicate other ways of introducing natural numbers." (³)

Although they soon came under fire, the "Recommendations" of 3 October 1968 have the twofold merit, firstly of seeing as a whole the process of modernising mathematics teaching from grades 1 to 13, and secondly of attempting to co-ordinate the efforts made by the various *Länder*. They were a first step forward, but that was all! So far as the training and further training of teachers was concerned, they made it clear that the following conditions were a pre-requisite for their implementation:

"1. If the modernization of mathematics teaching is to succeed, the training and further training of teachers in all types of school must be adapted to that end. Teacher-training establishments must ensure that the directives they give towards such modernization are properly syllabus-oriented, while remaining suitably open as regards teaching methods and practice.

2. . . .

3. The in-service training of teachers must be promoted in such a way that the modernisation of mathematics teaching can forge ahead on a wide front in all types of school. Here, co-operation between school and university is indispensable. Existing institutions in the *Länder,* such as the Institutes for the further training of teachers, one-week courses, holiday courses or seminars at the universities (contact studies), central offices for the teaching of mathematics and the natural sciences, teachers training colleges, distant-study courses, further-training courses as part of the dialogue between schools and universities or events organized by the professional associations – all these must contribute to modernizing the teaching of mathematics. Their contributions will bear fruit only if the results of their work are evaluated by professional conferences held in the schools and tested out in the classroom." (⁴)

Reform of mathematics teaching in the primary school: details and development

At the Ludwigsburg Conference on mathematics teaching in 1969, already mentioned, all the education ministers of the federal *Länder* urgently called for closer contacts between their respective areas, full exchanges of information, and co-operation between all those committees working towards new guidelines for mathematics teaching in different kinds of schools. Unfortunately these proposals were only partially put into effect. As a result, the directives and teaching programmes in the individual *Länder*, though based on similar premisses, reveal widely differing trends and planning concepts.

The KMK Recommendations (¹) have found a correspondingly varied echo in the directives for mathematics teaching in primary schools (³). We need say no more on this point here, however, as despite certain differences the basic problem of the further training of teachers has remained the same. We may confine ourselves to mentioning two matters:

1. Planning in the various *Länder* led to different datelines for introducing "new mathematics". Together with Hamburg, North Rhine-Westphalia was the last of the federal states to complete the "conversion": not until the 1973–74 school year did the new directives become compulsory there – and even this rule was restricted during the current year to the lowest primary grade.

2. The present debate in the FRG is so topical that, in the interests of clarification, and having regard to the importance of that debate in influencing teachers' motives for their own further training, it is worth while referring by way of example to the preamble to the directives for mathematics teaching in the primary schools of North Rhine-Westphalia (⁴). In that text we read as follows:

"The need for a reform of existing mathematics teaching is based on:

— changed social conditions, which have led to new educational and learning goals;

— recent findings of pedagogics and the psychology of thought, which demand a change in teaching methods;

— developments in the mathematical sciences, characterised inter alia by a structure-oriented build-up and an expansion of concepts and procedures in fields of application;

— progress in the didactic approach, leading to a multiplicity of new learning aids.

The essential changes in mathematics teaching in the primary schools relate to:

— general intentions: the aim of mathematics teaching is to contribute, in its own way, to the unfolding of the creative potential of the mind; firmly rooted in this, as an important partial goal, is the development of mathematical capabilities and knowledge;

- teaching syllabus: the inclusion of experience with quantities and relationships, geometrical concepts and the phenomenon of chance is intended to provide a broader basis for calculation and to bring out more general latent capacities necessary to an understanding of the world around us;
- ways of learning and ways of teaching: the use of new forms of instruction, teaching materials and presentation methods is designed to help the pupil to learn more than before from experience and to select his own ways of learning.

The changes provided for are not revolutionary: traditional ways of teaching are simply given a face-lift. Above all, account is taken of the ideas of Montessori, Kühnel and Johannes Wittmann, together with recent theories developed both at home and abroad. Hence "new mathematics" for the primary school is neither exclusively founded on, nor to be described outright as "the theory of sets" (7).

Finally, with regard to the importance of the leading structural concepts in primary-school mathematics teaching, an explicit reference is made to the "theory of sets" in its application to such teaching:

> "In the present context this term is understood as covering the basic structural concepts of quantity, relation, graphic representation (unequivocal co-ordination), and connexion, together with the attendant laws. The "theory of sets" in primary-school instruction is not something that has a separate existence alongside or over and above other subject-fields; in other words, it is not a subject in its own right. But this does not mean that on occasion (and, as the years go by, with an increasing demand for precision) the relevant concepts may not provide matter for instruction. It appears unnecessary to explain these concepts (sets, subsets, etc.) in the classroom. In appropriate situations they are trustingly made use of, practical action and pictorial representation being at first given priority. Any compulsion to express them precociously in words should be avoided. The day-to-day language used should not be "enriched" by technical terms until a real need for them arises. In the first two grades, as a rule, pictorial representation suffices.
>
> The basic structural notions should remain a principle pervading the instruction imparted. They represent a set of tools designed to serve the processes of mathematical learning." (8)

In-service training of mathematics teachers in primary and lower-secondary schools in the *Land* of North Rhine-Westphalia

The department responsible for mathematics and the natural sciences is Division 3 of the *Land* Teachers Training Institute. Locally known as the MNU, the department is situated at Recklinghausen.

The training problems raised by the KMK Recommendations were initially viewed by the Teachers Training Institute as follows: As the number of teachers affected by them in North Rhine-Westphalia was very large (over 50,000 in the *Grundschulen* and *Hauptschulen* together), a graduated system of further training was the only possible solution.

Project for graduated in-service training

Accordingly, the senior supervisory authorities for schools, namely the six *Regierungspräsidenten* in Aachen, Arnsberg, Detmold, Düsseldorf, Cologne and Münster, were requested to nominate certain persons with some previous knowledge of the question and known to have given special attention to the theory of mathematics teaching and to have presided over conventional working parties in this field at regional education-committee level. The colleagues thus selected would be intended to act as regional "multipliers", one for each supervisory district.

For those appointed (teachers from primary schools and secondary classes 5 and 6) a series of training events took place in 1970 and 1971 under the general title "Modernisation of mathematics teaching in grades 1–6". The aim of these meetings was to make an extensive and planned contribution, within the possibilities of the Teachers Training Institute, towards providing the teaching corps with appropriate information prior to the coming into force of the KMK Recommendations and hence towards putting into practice the proposed guidelines for grades 1 to 6, covering the following items of the syllabus:

1. Sets and set operations
2. Sets of natural numbers, operations
3. Dimensions
4. Basic geometrical concepts
5. Figures and positional systems
6. Divisibility and fractions
7. Sets of non-negative rational numbers, operations.

It was planned that each participant, as a potential "multiplier", should attend two one-week sessions with an interval of about a year between them.

The first study sessions — there were four, because the supervisory authorities cover areas of differing size — were attended by 60 teachers each in accordance with the following timetable:

16—25 March 1970 (participants from the Düsseldorf area)

28 Sept.—2 Oct. 1970 (Münster and Detmold areas)

11—15 January 1971 (Arnsberg area)

6—10 Sept. 1971 (Cologne and Aachen areas).

The general scheme was to provide technical information, i.e. a basis of theory, against a didactic background, with hints for day-to-day teaching practice. Each participant was given extensive practical exercises to familiarise him with the New Mathematics and so enable him, after further personal study, to pass on his findings in the form of useful proposals to colleagues, to whom he would later be available for help and advice.

Sixty participants were invited to each session, so that there would be one representative from each supervisory district. Their selection was a matter for the *Regierungspräsident* or the local school boards and could not be decided or influenced by the *Land* Teachers Training Institute. This method was necessary, since the Institute is not in a position to judge in every case whether potential candidates fulfil the required conditions, or to form an accurate picture of the teacher situation in individual schools — and also because an official journey (attendance at a conference being classed as such) can be ordered, under civil service regulations, only by the competent higher supervisory authority, i.e. in this case the *Regierungspräsident*.

All four sessions of the first series were planned in the same way, the programme being as follows:

Monday:

3—3.15 p.m.	W. Böddeker — Recklinghausen: Address of welcome: introduction
3.15—4.45 p.m.	Prof. Dr. H. Winter — Dortmund: "Necessity, rationale and possibility of modernising the teaching of mathematics. The KMK Recommendations and the new directives" (*) Discussion

Tuesday:

9—10.30 a.m.	Prof. Dr. P. Sorger — Kiel: "Introduction to the theory of sets"
10.30—11 a.m.	Coffee break
11—12.30 p.m.	Dr. Sorger's paper (contd.) Discussion
3—5 p.m.	Working groups: exercises and examples to illustrate and expand the problems raised at the morning session

Wednesday:

9—10.30 a.m.	Dr. P. Sorger — Kiel: "Further introduction to the theory of sets"
10.30—12.30 p.m. 3—5 p.m.	as on Tuesday

Thursday:

9—10.30 a.m.	Dr. H. Winter — Dortmund: "Modernisation of mathematics teaching: various possible views on methods of instruction"
10.30—11 a.m.	Coffee break
11—12 noon	Dr. Winter's paper (contd.) Discussion
2.30—4.30 p.m.	Dr. Simm (Duisburg): "Ways and means of modernising maths teaching in classes 1—6. Examples of modern instructional processes" Discussion

Friday:

9—10.30 a.m.	Dr. Simm — Duisburg "Examples of modern instructional processes" (continued)
10.30—11 a.m.	Coffee break
11—12.30 p.m.	Concluding discussion

The foregoing details give some idea of the time devoted to the various aspects of this session.

The brief opening address outlined the purpose of the meeting and its basic plan. The introductory report, starting from various premisses, then explained the reasons why a reform of mathematics teaching in the primary schools was necessary (*); different possibilities were discussed, and there followed a detailed debate on the KMK "Recommendations" of 3.10.68 and the reasons and conditions set forth therein.

On the Tuesday morning participants were introduced to the treatment of quantitative algebra in the primary school:

Different forms of illustration:

1) Venn diagram

2) Logical games (Boolean algebra)

3) Sorting machines (transposing of truth tables)

4) Karnaugh diagram (Caroll diagram)

Applications to exercises relating to real-life situations.

The Wednesday morning meeting was devoted to more abstract treatment of quantitative algebra and propositional calculus:

Introduction to conjunction (1)
alternation (v), subjunction (→)
negation (>) by means of truth tables
introduction to quantity relations (∩, U) via truth tables
connection between fractional relation (C) and subjunction (→)
matrices and solutions
evaluation of assembled propositions
a) Truth tables
b) Evaluation by substitution and reduction
Arrangement of rules
Transposition of evaluation procedures to truth tables.

On the afternoons of both days participants had an opportunity of working in small groups, under the leadership of experienced colleagues, on exercises and examples illustrating the problems dealt with in the morning, in the light of their relevance to teaching practice.

On the Thursday various practical ways and means of teaching the New Mathematics were presented, the different standpoints also being compared with the 1968 "Recommendations" and with conventional methods of mathematics instruction. There was, in addition, a thorough debate on the resulting general views expressed in regard to the teaching aspects.

The Thursday afternoon and Friday morning sittings dealt with illustrative examples of possible or already tested teaching methods consistent with the desired reforms. In the concluding discussion the participants all agreed on the urgent need for further information; a further week's study session was in any case already planned.

On the basis of their own teaching experience and of in-service training courses which they themselves subsequently were to organize in the different regions, the participants proposed that the next sessions should take place in just under a year's time and should be devoted to additional exchanges of information, primarily on practical teaching aspects.

Although it was noticeable that some participants were sceptical about their functions as "multipliers" — the reasons for this attitude may be found in the sometimes insufficiently careful choice of participants by the supervisory authorities, as well as in the admitted difficulties of the entire exercise, designed as it was both to provide professional information and to transpose didactic method and theory into practical work in the classroom — the four study weeks of the first series nevertheless were well received by the majority of those attending.

The four one-week courses of the second or follow-up series were held as planned, namely:

15—19 February 1971 for the first group (Düsseldorf)

4—8 October 1971 for the third group (Arnsberg)

6—10 December 1971 for the second group (Münster and Detmold)

24—28 January 1972 for the last group (Aachen and Cologne).

The timetable was the same for all 4 weeks, viz:

Monday:

3—3.15 p.m.	W. Böddeker — Recklinghausen: Address of welcome; introduction
3.15—5 p.m.	Prof. H. Houben — Aachen: "Groups in primary school mathematics teaching" Discussion

Tuesday:

9—10.30 a.m.	E. Hollmann — Hannover: "Mathematics teaching in school years 1 to 3, with special reference to the treatment of relations and games of logic"
11—12.30 p.m.	Mr. Hollmann's paper (contd)
3—5 p.m.	Mr. Hollmann's paper (concluded); discussion

Wednesday:

For the morning session two demonstration lessons had been arranged.

3—5.30 p.m.	Prof. Dr. H. Winter — Dortmund: "Mathematics teaching in grades 4 to 6 of the Grundschule/Hauptschule. New aspects of concrete arithmetical operations *(Sachrechnen)*" Discussion

Thursday:

	Prof. Dr. P. Sorger — Kiel:
9—10.30 a.m.	a) "Different methods of teaching the basic concept of number"
11—12.30 p.m.	Continuation of paper; discussion
3—4.30 p.m.	b) "Possibilities of performance control in primary school mathematics instruction. Practical achievement of learning goals, illustrated by teaching processes selected as examples" Discussion

Friday:

9—10.30 a.m. and 11—12.30 p.m.	Continuation of paper b); discussion
2—4 p.m.	Concluding discussion

Where possible, the demonstration lessons were divided among three groups, i.e. there was a choice of six, 3 in the first half of the morning and 3 in the second half, so that each participant could attend two of them. The subjects with which they dealt were, on the one hand, "conventional" (e.g. percentages and interest calculations, equations and inequations), the idea being to show clearly how such classic themes can be treated from a modern standpoint and with up-to-date methods.

On the other hand, in classes that were being taught in the "modern" way — in the sense of the KMK Recommendations — care was taken to see that alternatives were offered as regards the manuals introduced, whence the varying possibilities of carrying out the reform were demonstrated. (In this connexion, see the two analyses by Griesel and Hollmann (10) and Griesel (11) of the current teaching-manual situation with reference to the modernisation of mathematics teaching in the primary school.)

The concluding discussion showed that about 60 % of the participants were already exercising their "multiplier" functions — whether by running working parties for student teachers, by starting voluntary study groups meeting at varying intervals and with varying numbers of participants, by holding regular professional conferences for one or more schools or by advising colleagues in their area in regard to literature and teaching aids.

Many expressed the wish for further information, on topics such as geometry and topology, wider aspects of numeration, elementary combinatory theory and probability calculation, problems of mathematics teaching in the "promotional" grades (school years 5 and 6), modernisation of secondary-school mathematics teaching and special "study weeks" for teachers in experimental primary schools.

These demands, essentially of a technical nature or related to syllabus content, showed the need for further sessions. Behind the call for more information could, however, be discerned, or at least glimpsed, the still unresolved problem of the proper translation into practical terms of the didactic methods discussed in their theoretical garb. Some of the proposals expounded were in favour of a straightforward transfer of the technical and scientific information to the ambience of the classroom; others advocated the retention of conventional teaching methods to be illumined only here and there by the lamps of the "New Mathematics".

It became clear that the real problem lay, not in the provision of technical information, but in changing the teachers' approach, in enabling them to teach mathematics in another manner. In a word, their accustomed attitude to "schoolteaching" needed a thorough shake-up.

This problem, it was seen, was in no sense more than partly solved, even for those who were to act as "multipliers". To that extent the assumption that those invited to attend these sessions were appropriately motivated and sensitised showed itself to be wide of the mark. Moreover, some of the participants, feeling that there was too great a demand on their energies, were no longer prepared to spread the gospel by taking upon themselves the organisation of regional in-service training arrangements.

Planning of "supplementary training" of "multipliers"

Other ways of imparting information to these key figures were, however, under consideration. It all began when the higher supervisory authorities gave their approval, for the Autumn of 1972, to a fresh selection and appointment of suitable persons.

To clear the ground for further sessions, a group of interested professors and lecturers from teacher-training colleges, in June and October 1972, met together to thrash out plans for the training of "multipliers". To avoid waste of time, it was important that the next sessions be attended by teachers already possessing adequate technical knowledge. The minimum qualification to be required of potential candidates by the authorities, it was proposed, should be approximately the degree of mathematical knowledge necessary for passing the first examination for primary and secondary school teachers. Before acceptance, details should also be given of whether, and if so where, the teachers concerned had already taken an active part in the further training of others.

After some discussion, it was agreed that two half-yearly courses of 30 hours each would be allotted to the "supplementary training of multipliers". This period was regarded as adequate for a minimal programme. The courses should offer scientific information on modern mathematics coupled with considerations of teaching theory and proposals for classroom practice. The need for scientific information was felt by all to be paramount: if it were omitted, there was a risk that the participants would be unable to get away from the teaching models suggested. Some degree of theoretical information is also necessary to ensure uniformity in the knowledge acquired, but it must not be an end in itself: rather should it be selected with an eye to teaching practice and backed up by examples from the curriculum. Broadly speaking, the courses would contain the following themes:

Sets and set operations, propositional calculus
Relations, particularly in diagrammatic form
Structure of sequences, particularly groups
Isomorphs
Cardinal numbers; basis of calculation with natural numbers
Elementary combinatory processes.

The question whether other approaches to natural numbers could be adopted and how far traditional arithmetic should be taught gave rise to differing opinions. It was agreed, however, that participants' wishes should be taken into account when the content of the courses was determined, and that some attempt should be made at differentiation in accordance with the amount of previous knowledge necessarily possessed. The course should be preceded by a two-day introductory meeting, when those in charge – lecturers from Teachers Training Colleges – should be informed about the status and wishes of participants and, in return, give them a bird's-eye view of the themes to be presented and some idea of their treatment.

Distant-study course on "Mathematics for Primary-School Teachers" and its impact on the further training of such staff

Such was the position at the end of 1972, when it was overtaken by technical difficulties on the part of the supervisory authorities and by the fact that in the meantime the distant-study course "Mathematics for Primary-School Teachers" (¹²) had become available.

Object of the course

The synopsis of this course, in its preface (¹³), describes its aims as follows: "This distant-study course is intended to serve the further training of teachers by enabling them to impart "modernised mathematics teaching" as laid down for the period beginning with the 1972/73 school year in the Recommendations of the Conference of Ministers of Education held in 1968. The majority of today's primary-school teachers do not, by their training, possess the necessary previous knowledge that will fit them to carry out the KMK Recommendations effectively. Experience to date shows that only in exceptional cases is it possible for a working teacher to acquire this knowledge on his own initiative or from occasional attendance at further-training sessions."

The preface states expressly once again that it is not enough to add a few new "mathematical" items to the syllabus, to be acquired by traditional methods. Rather should these items simply provide the framework for developing mental activities and the productive stimulation of thought. The urgent need for further training is stated in the following terms: "If these efforts are to bear fruit, it is vital that the teacher should not only speedily acquire the new mathematical material, but should learn to think in mathematical terms: he must have a fluid and open approach to new questions and be prepared for undogmatic work with children who, in sizing up unfamiliar situations and concepts, may often be his equals or even his superiors. This especially demands of the teacher that he rid himself of any inward aversion to mathematics, such as many adults have felt since their own schooldays – even teachers are not exempt from it. The foregoing makes clear the need for efforts to ensure the further training of teachers to a degree that is manifestly very much underestimated in the KMK Recommendations." (14)

Although this last contention certainly corresponds to the facts, it is still far from sure – particularly in the light of experience to date – whether and to what extent the distant-study course can achieve its avowed aim. Apart from this, however, its great merit lies in the fact that, for the first time ever, the printed synopsis of the course makes available material that can be put to practical use.

Structure and content of the course:

The various units of study are grouped together in "packages", as follows:

	E 1	Sets and their presentation
	E 2	Sets operations
1st package	E 3	Cardinals, addition and subtraction
	E 4	Multiplication and division
	E 5	Positional systems
	E 6	Matrices and their degree of accomplishment
2nd package	E 9	Relations, in particular order and equivalence relations
	E 10	Dimensional fields
	E 11	Algorisms, written calculation procedures
	E 12	Operator games
	E 13	Graphs and their concatenations
3rd package	E 14	Sequences and their properties
	E 15	Groups
	E 16	Simple geometrical concepts
	E 7	Theory of divisibility
	E 8	Primary numbers, breakdown of primary factors

4th package	E 17	Fractions (positive rational numbers)
	E 18	Whole numbers, rational numbers
	E 19	Simplest functions and their applications in "concrete arithmetical activities" *(Sachrechnen)*

The addition of one further item

E 20 Probability calculation

is planned.

The first package contains the essentials of instruction during the first school-year. Study of the remaining items may then take place as a corollary to the teaching sessions, the first package being intended to cover school-years 1 and 2, while the 4th comprises the main focal points of years 5 and 6. The two other packages are systematically constructed and the chronological order in which they are arranged does not necessarily follow the usual classroom practice.

Each "study kit" is divided into

a technico-scientific text (F)

a didactic and methodological text (D)

work-sheets (A) and

solutions to the work-sheets (L).

It is not intended that the texts should be worked through in the order F-D-A-L, nor is this the order in which they are arranged in each "kit"; rather does each chapter begin with an F-text, to which the appropriate D-text is appended. The work-sheets, which are detachable, and the booklet containing the solutions, are for practical use as they stand.

The work-sheets are linked with the previous knowledge generally presumed to exist. They serve a dual purpose: (a) preparation for study of the F-text, i. e. motivation for the treatment of technical subjects, and (b) to round off and deepen acquired knowledge and help apply it to other problems.

The F-text, which can be read as a self-contained unit, "places the mathematical material in a realistic context, restricting itself to a brief pragmatic treatment of the most important aspects. The aim is to give the teacher the indispensable background knowledge underlying modern maths teaching, independently of any special methodology ([15])".

"The D-text adds to the mathematical subjects treated indications of their significance for the teaching imparted (theoretical reasoning), as well as proposals for motivations, illustrations, jargon and types of exercise. In so doing it also illustrates the F-text and gives the reader additional motivation for his technical work. Predominantly, however, it is designed to provide help and suggestions for translating into classroom practice the knowledge acquired from the work-sheets and the F-text." ([16]) But no attempt is made to arrive at a cut-and-dried teaching theory. The work-sheets and F-text are not, by their content and mode of presentation, intended for direct use in the classroom, and although, so far as possible, the D-text is couched in the language used by the teacher in front of his pupils, detailed suggestions on method cannot, in the nature of things, be given.

As regards the time to be taken over the course, provision was initially made for each item to be accorded four weeks' private study, to be accompanied by two seminars of 2 hours each. This resulted in a total period of 2 years for the course as a whole.

Planning and running the course

By agreement with the school boards the "supplementary training for multipliers" was postponed and preference given to the institutition of distant-study courses on "mathematics for primary-school teachers"; the "multipliers" under consideration would, where practicable, be appointed directors of studies for the distant-study courses.

The over-all planning was as follows:

Each director of studies would be responsible for a course usually grouping about 30 participants from a given region: i. e. he would hold the accompanying seminars and "mother" the participants. Interested and well informed teachers, especially those who had taken part in the two "study weeks" described above and who were regarded as potential "multipliers", were proposed by the supervisory authorities as directors of studies and appointed by the *Land* Institute at a set fee. Thus, on the one hand, the amount of information possessed by these potential "multipliers" was exploited, and, on the other hand, the directors of studies were themselves prepared for their directorial duties by means of a "Meta" course, comprising between 20 and 30 participants and given by subject-heads from the departments of teaching theory, method and technique at secondary-school teachers training colleges.

The courses for the directors of studies began with a full-scale conference lasting two days and were continued in the form of one-day sessions devoted to each study item in turn, with the object of familiarising the participants with the subjects they would be introducing in the regional distant-study courses and affording them general assistance in the running of these. Problems arising during such courses were discussed, up-to-date information on the theme "Mathematics in the primary school" was passed on, suggestions and experiences were exchanged. Voluminous tasks, such as the analysis of school text-books, were divided among participants, and it proved possible to enlist the services of two professors to act as advisers.

Over and above the regular one-day sittings special questions primarily concerned with teaching practice were dealt with, where necessary, at ad hoc meetings lasting 3 days, when detailed indications on methods were also given.

Participation in the distant-study courses was voluntary, though strongly recommended to all teachers in charge of a first-year course. The course literature was, naturally, sent to them free of charge. No "time off" from lessons was granted, however.

It soon became apparent that the time needed to cover the material relating to one item (literature and work-sheets) was not 4 but at least 6 weeks. This put the total duration of the course up to 3 years. This 6-week rhythm has so far been maintained: as a rule one afternoon every 3 weeks is devoted to an accompanying seminar, and every 6 weeks a fresh item of study is distributed. The course for the directors of studies follows the same rhythm. A system whereby the directors are two items ahead of their own regional courses has proved its worth.

From January 1973 to February 1974 eight "Meta" courses for directors of studies were held, in collaboration with the school supervisory authorities, for a total of 250 directors. Four of them took place in the area covered by the *Regierungspräsident* of Düsseldorf, and one each in the areas of Cologne (17), Münster, Detmold and Arnsberg. This means that 250 regional distant-study courses were started, with a total of some 7,200 participants. If this last figure is compared with the total number of primary and secondary school teachers in *Land* North Rhine-Westphalia, it will be seen that a bare 15 % took part in the courses. In the case of primary teachers, however, since their number is disproportionately greater, the percentage must be correspondingly higher.

Criticisms; experience to date

As all these courses are still in progress, no final judgment of their success can be made. But a few findings may none the less be mentioned:

(a) All those taking part find the distant-study courses a considerable burden.

(b) Since no "time off" from lessons is granted, only some teachers are willing or able to take on this burden, which after all is a voluntary commitment. This tendency is all the greater now that the courses take 3 instead of 2 years.

(c) The course is not self-motivating – indeed, it may be asked whether any mathematics course ever can be. As a result, the literature distributed has often been worked through inadequately or not at all by the students.

(d) The students' motivation was also adversely affected by the fact that the question of issuing a certificate at the end of the course was never cleared up.

(e) One serious disadvantage of the course lies in the fact that insufficient advice is given on method (10). This detracts seriously from its value. The normal teacher likes to have teaching hints that can be copied – practice based on theory: in other words, practical methods to adopt.

(f) Thus it now lies with the directors of studies to give hints on teaching practice during the accompanying seminars, whose "accompanying" functions must give way to an important new task not performed by the distant-study course, namely to change the teachers' approach – to enable them to dispense mathematics instruction in a manner to which they are not accustomed: in short, to sensitise them to a changing situation (12). (Cf. "Object of the course", page 119 above.)

In order to make up for this deficiency, the seminars of some courses were expanded as the need arose to whole-day sessions, the morning half beginning with demonstration lessons in modern mathematics with the aim of putting the participants into closer touch with practical realities.

(g) Finally, many directors of studies felt themselves to be, and in fact were, overloaded by the responsibilities involved, despite the information and help they received, and gave up the assignment after working through the 1st package.

Thus, after working through package 1 (items E 1 to E 5) — corresponding to three-quarters of a year's participation in the distant-study course — about 30 % of the students gave up, for the reasons stated above. There were fewer defectors between packages 2 and 3. That 30 % should have dropped out after the 1st package is bad for the image of mathematics reform, for the impression may be created that the reform of mathematics teaching in the *Grundschule* essentially consists of introducing elementary concepts of the theory of sets and nothing more (20).

Taken as a whole, however, the courses must be regarded as beneficial, as regards both the quality of the results obtained — participants were effectively put in a position to teach modern mathematics — and also, especially, the large number of teachers with whom contact was established as it could hardly have been in any other way.

It will be important in planning future courses to settle the question whether and how far — adequate "time off" being allowed — attendance can be made compulsory.

In-service training at Teachers Training Colleges

All these establishments were ordered to introduce arrangements for the further training of teachers in new mathematics, and appropriate courses were in fact held in the years 1970 to 73. They consisted mostly of lectures, with exercises, taking up two hours a week in term-time. The stress was on technical information against a didactic background, the main subjects being sets, relations, graphs, fields of numbers and their extension, positional systems, dimensional fields, geometry and graphic topology. Some of these arrangements were intended especially for teachers; some — the smaller part — were directed at students, though practising teachers could also profitably attend these.

The teachers were, however, somewhat reticent to accept this offer. In most cases they failed to do the prescribed exercises, pleading a variety of reasons, especially shortage of time — a powerful argument, since no time-off from their teaching duties was granted. This problem always loomed large. The teachers' own motivation varied. There was a widespread view that mere attendance at the lectures must suffice. Recognition of the need for independent work and the willingness to "do a bit of maths" on one's own could not be taken for granted and could, indeed, only rarely be awakened.

Accordingly, the lectures gradually ceased to be a means of imparting technical information as a basis for didactic analysis, and more and more took on the function of an accompaniment to classroom instruction, sometimes with direct reference to a particular mathematics text-book for use in schools.

Mention should be made here of similar arrangements made by further-training establishments set up under the responsibility of individual towns and associations. About the beginning of 1974, when there was a wave of criticism directed at "the theory of sets in the primary school", a strong tendency towards disaffection was noted: lectures of this kind ceased to be attended, although it is of course possible that the distant-study course "Mathematics for Primary-School Teachers" is regarded as a better alternative.

Special further-training arrangements for the heads of certain mathematics departments

Since 1969 the Recklinghausen branch (MNU) of the *Land* Teacher-Training Institute has been holding regular study-weeks for heads of the mathematics departments at the seminars which take place at area *(Bezirk)* level for primary and secondary teacher trainees. The study-weeks are so organized that each head of department will have attended one or two every year, and been systematically and comprehensively informed about the reform of mathematics teaching in the *Grundschule*, as regards both the technical background and the transposition of teaching theory to classroom practice (²¹). They are thus in a position to act as tutors to candidates for the teaching profession who have just passed their first (theoretical) public examination on leaving their training colleges, and to initiate them into classroom practice. They help to give the fledgling teacher's self-assurance a healthy shake-up and to destroy the "schoolmarm" image of the conventional maths teacher: it is they who are the real reformers and pioneers of the New Mathematics — even though they may at first appear to be only "backroom boys".

An enquiry carried out among these department heads in 1971 showed that in their experience only about 10 % of the teacher trainees were capable, on the basis of their studies, of putting the reform of mathematics teaching into practice. The percentage is certainly a little higher today, but there has been no spectacular change in it.

The reason for this does not lie in any inadequate provision of lectures at the universities – at any rate not for some years past now. It is rather to be sought in the instruction given to date in the teacher-training colleges.

Alongside the activities mentioned here, there has of course been a plethora of other arrangements for the further training of mathematics teachers in the *Grundschulen* and *Hauptschulen:* there have been special projects for certain small and restricted groups — and similar schemes at regional and local level. These were launched by the *Land* Teacher-Training Institute and similar bodies run by the Churches, associations and individual towns, by the school boards and, last but not least, as a result of private initiative. We cannot go into details of all these arrangements, which are doubtless found in similar form in all the Federal *Länder*. There is ample proof that official efforts to enable teachers to master the required reforms have benefited from their assistance, even though it has varied in degree from one region to another.

Training of teachers for *Grundschulen* and *Hauptschulen* in the Federal Republic

With some differences as between the federal *Länder*, teacher-training courses at these levels are as follows:

Phase 1: Studies at a teachers training college, polytechnic *(Gesamthochschule)* or university, and

Phase 2: preparatory service.

Ad 1: The theoretical studies cover 6 semesters (3 years) and comprise courses in appropriate educational theory and in the subject(s) to be taught.

Ad 2: Preparatory service takes place at a seminar held on an area *(Bezirk) basis,* aimed at providing a solid fund of technical knowledge suitable to exercise of the profession: it includes practical work by the students, lasts for 1 year and concludes with the second public examination (pedagogical aspects).

The heads of the various subject departments act as tutors to the candidates.

Phase 1: Studies

Training in the different higher education establishments (Hochschulen) is, with minor variations, the same in all the *Länder*.

A would-be primary- or secondary-school mathematics teacher in North Rhine-Westphalia, if he selects mathematics as a compulsory subject *(Wahlpflichtfach)*, attends during his 3-year course a total of

36 hours of lectures or exercise-sessions in term-time, receiving technical information on his subject, as well as hints on the theory and methods of teaching. But not more than 10 % of all students opt for mathematics as a main subject: about 30 % take it is a supplementary special subject *(Stufenschwerpunkt)*, comprising a total of 12 hours' instruction, i. e. 2 hours per week per term, when primarily, if not exclusively, direct advice is given on method and teaching practice. It may be questioned whether this is a valid foundation for independent teaching of mathematics, especially in view of the impending reform.

Conditions in the other *Länder* are similar. For example, at a teacher-training college in Baden-Württemberg the student may take mathematics as a main subject, when he will attend a total of 40 hours' instruction in term-time during his 3-year course — 20 hours of technical instruction in his subject and 20 hours of teaching theory, i. e. 6 hrs per week over 4 terms and 8 hrs per week over the other 2. If mathematics is taken as a subsidiary subject, the instruction comprises 20 hrs spread over the 3 years in term-time — 12 hrs subject-instruction and 8 hrs teaching theory, i. e. about 4 hrs per week for 4 terms and 2 hrs per week for the other 2. In the winter semester, 8.6 % of the students intending to become primary-school teachers chose mathematics as a main subject while 6.9 % chose it as a subsidiary subject; of those aiming at the *Hauptschulen* the figures were 6.1 and 5.9 % respectively. In all, therefore, 14.7 % decided for mathematics as a main and 12.8 % as a subsidiary subject. The same objection probably applies to the choice of mathematics as a subsidiary subject as to its choice as a supplementary special subject *(Stufenschwerpunkt)*.

At a general establishment of higher education in Hessen 15 % of the students chose mathematics as a main subject (38 hrs in this case over the three years). The following is a typical example of a programme of studies:

1st term:	Basic mathematical concepts	4 h
	Supplementary information for new entrants' class	2 h
2nd term:	Basic algebraical concepts	4 h
	Concept of number	2 h
3rd term:	Theory of mathematics teaching	4 h
	Geometry	2 h
4th term:	In new entrants' class	4 h
	Dimensions	2 h

5th term:		4 h
	Number fields	2 h
6th term:	Teaching of maths. to grades 5 & 6	4 h
	Seminar	2 h
	Seminar on teaching theory	2 h

Practical training during the study phase is restricted in North Rhine-Westphalia to a period of general teaching in a school *(Blockpraktikum)* and a practical course to illustrate the theory of mathematics teaching *(fachdidaktisches Praktikum)*.

The *Blockpraktikum* takes place during the vacation: the student is attached to a school for 4 weeks and gives lessons on his own. These are not tied to a particular subject.

Only students who have chosen mathematics as a main subject attend the other course. For two hours a week, spread over 2 terms, they are taken by a tutor as a group to visit classes in progress, and subsequently discuss problems of planning, preparation and conduct of mathematics instruction. Such courses are found in all the federal *Länder*.

Although the differing situations in the *Länder* make it difficult to give precise details of conditions and requirements, it may be said that in general about 10 % (maximum 20 %) of the students receive about 40 hours' mathematics instruction during their six terms, which may be considered adequate preparation for teaching mathematics in the *Grundschule*.

Phase 2: Preparatory service

Practical training proper takes place during "Preparatory service", which takes the form of attendance at a seminar (held at *Bezirk* level) for candidates for teaching posts in *Grundschulen* and *Hauptschulen*. This service at present lasts for a year: in some places its extension to one and a half years is being envisaged.

All candidates are obliged to attend a course in general pedagogics, based on practical realities, and are split into groups for their own particular subjects. In the case of mathematics, the groups consist of candidates who during their period of studies selected mathematics as a main subject, with the addition of those who took it as a supplementary special subject (*Stufenschwerpunkt*) if they so desire. Since every student takes two such "supplementaries", about

50 % of those who included mathematics as one of them join the maths group in the seminar. Each group consists of some 10 candidates under the supervision of a subject head (*Fachleiter*).

The group holds a two-hour meeting once a week, to discuss questions of subject teaching and method: technical questions are dealt with only in so far as they may need didactic analysis. Gaps in technical knowledge cannot, of course, be filled in. Every 3 or 4 weeks the meeting is replaced by a group visit to a school, when candidates can familiarise themselves with new teaching methods in operation.

The conclusion to be drawn is that in this way about 30 % of all candidates receive practice-related training in mathematics teaching; should the occasion arise, however, all of them must later be able to teach mathematics in primary school.

In-service training in the individual *Länder*

In-service training arrangements in the city states of Berlin, Bremen and Hamburg have the advantage that they present no difficulties of an organisational or technical nature, such as travel, board and lodging, etc. Everything can be arranged on a "regional" level, which ensures greater flexibility as compared with the other *Länder* — quite apart from the smaller numbers involved.

Berlin

Half-year courses

Preparation:

In the summer semester of 1969 an experiment took place, in co-operation with the Teachers Training College, whereby a group of primary-school teachers having mathematics as their main subject were invited to a course whose object was to familiarise them with the problem of indoctrinating other colleagues into the teaching of the "New Mathematics".

Implementation:

In the winter semester of 1969/70 courses were started in all the administrative areas (*Bezirke*) on "New Mathematics Teaching in the Primary School". Each course, to be attended by at least 20 primary teachers, consisted of 24 weekly two-hour periods spread over half a year, and designed to introduce the participants to the content and new didactic approach of the "New Mathematics". To date about

3,000 have taken part in roughly 100 courses, or more than 50 % of all primary teachers in Berlin.

Relatively speaking, this is a high turn-out. It has not, however, been without its problems. After attending the course a number of teachers had the impression "that the pre-numerical field of mathematics teaching in the *Grundschule* represented an additional burden of material. This was part of the teething troubles of such a comprehensive training measure and could to a great extent be put down to the meagre practical experience of the tutors. If their abilities were wrongly assessed, the reason lay partly in their well-meaning efforts to pass on to the participant teachers, alongside didactic advice, a fund of knowledge that was above the heads of their pupils" ([22]).

Other arrangements:

Since 1970 important points of mathematics teaching, and where necessary closely related themes, have resulted in 14 one-day courses of further training designed to clarify certain ideas held by teachers and to demonstrate the relevance of "New Mathematics" in day-to-day teaching.

Distant-study course on "Mathematics for primary-school teachers":

At the beginning of 1974, 150 teachers were taking part in this course, which is on much the same lines as its counterpart in North Rhine-Westphalia ([23]). An increase in the number of participants is planned.

Bremen

Here, and in Bremerhaven, the further training of teachers is in the hands of the "schulpraktischen Instituten" ([24]).

Two-year courses:

In recent years the emphasis has been on long-term training arrangements intended to make participants capable of teaching "new mathematics" officially in the primary school. The courses, which have been running since 1970, consist of a 2-hour lecture every week, or 10 per quarter, and a 30-hour seminar lasting a week on a whole-day basis. There are 150-225 participants per course; a seminar-week is attended each time by 30 teachers, who are of course freed from their duties for the purpose — during the remainder of the course they are granted 3 free periods per week. The courses are to be concluded by 1976, by which time about 1000 primary-school teachers will have participated, or 75 % of all such staff in *Land* Bremen.

The lectures deal primarily with technical information, so far as it appears relevant as a background to mathematics teaching in the *Grundschule*. Themes from the following areas are treated: propositional calculus, sets, relations, diagrams, bonds, arithmetic, structures and elementary geometry.

The seminars, on the other hand, are devoted more to the translation of theory into classroom practice: attention is given to teaching programmes, manuals for mathematics teaching, games, and teaching aids. Unfortunately no demonstration lessons are held.

Other arrangements:

As in all the other *Länder*, there are local working groups and individual schemes as an accompaniment to teaching practice.

Hamburg

Here the responsible body for further training is the advisory office for mathematics attached to the *Institut für Lehrerfortbildung* ([25]).

Four-week courses

Since 1967, during the period from February to June, courses lasting 3-4 weeks have been held on a whole-day basis, participants being freed from their duties in order to attend. These arrangements have proved to be especially possible in Hamburg because in 1967, when the "running-in" period for new school-entrants was changed so as to start in the autumn instead of after Easter, Hamburg did not follow the other *Länder* in introducing short school years: i.e. from 1967 to 1975 school-leavers have been dismissed at Easter and new entrants received in the autumn, so that there is a surplus of teachers from February to June.

These courses are run by part-time lecturers, subject heads (*Fachleiter*) from the seminars for candidate *Gymnasium*-teachers, or by training-college lecturers and practising schoolmasters. They last from 90-120 hours. The subjects to be treated are defined in the directives issued by the Hamburg education authority ([26]), which state what innovations are to be introduced and in what light the conventional forms of teaching should be viewed. The general trend is described as follows: "It is not intended that mathematical structures shall form a new field of material alongside the traditional curriculum. Rather should the aim be to render mathematical relationships clearer through use of the new concepts and to extend their field of application. Teachers should therefore endeavour to integrate the new forms with the old, to the greatest possible extent" ([27]). Some of the manuals intended for in-service training courses have already been published ([28]).

The first week of the courses is devoted to introducing elementary mathematics — sets, relations, diagrams, number —, the lecturers being professional mathematicians, mostly from *Gymnasium* circles. A few initial doubts proved to be without foundation, as the lecturers found no difficulty in getting on the same wavelength as primary-school teachers in regard to modes of thought and expression. Although in the following weeks further material information was given, the stress was then placed principally on actual teaching aspects, questions of the planning and analysis of instructional methods being also discussed. All participants were taken daily as a group to watch a class in progress, sometimes after common preparation and in every case with subsequent common analysis of what had been observed.

Courses in term-time

Eight to ten two-hour training sessions are also offered as an accompaniment to teaching. They are organized on an education area level and provide a follow-up to the subjects dealt with at the 4-week courses. They constitute a direct complement to teaching and frequently relate to the text-book in use in the schools.

An annual 10-day "summing-up" course (*Klausurtagung*) has also been held since 1970, on a school-area basis, for teachers who have benefited from other in-service training arrangements. General problems of such training are discussed, and in addition special attention has been given each time to a particular subject. Although this institution has proved useful, it is felt that this problem cannot be solved by recourse to "multipliers" alone.

Since it is thought necessary to continue the 4-week courses until 1980 and there will no longer be a teacher surplus after 1976, efforts are being made to create additional established posts, so that a system of "supply teachers" for those undergoing in-service training can become an accepted feature.

Baden-Württemberg

For all primary-school teachers, except those who in accordance with the new training system took mathematics as a main or subsidiary subject with special reference to primary schools, compulsory in-service training in "New Mathematics" was started. Here there was a problem of some size if 20,000 teachers were to be prepared for their new tasks in 3 years, and it could be solved only by recourse to a system of "multipliers".

Blueprint for "multipliers"

In 1970 and 1971 three hundred committed and experienced teachers were brought together at the Teachers Training Colleges and State Academies ([29]) for "contact courses" aimed at producing "multipliers". The system involved 10 separate study days and two or three one-week sessions: the instruction given, as regards both subject-matter and teaching theory, was such that from the 1971/72 school-year onwards the participants were themselves able to run study sessions each attended by 25 other teachers. For want of any better background literature the "multipliers" were all put through the radio course "Funkkolleg Mathematik" ([30]): this had certain specific effects, in that it made them more aware of concepts than of problems.

Compulsory in-service training arrangements

Practical arrangements

In the 1971-72 school-year about 9,000 teachers attended these courses, including all who in 1972-73 had to take over a new entrants' class. In the following year the total was about 7,000 and in 1973-74 it was some 5,000. The outcome of the "multiplier" system was that no primary-school teacher is any longer without this supplementary training in the new mathematics.

Content

There were 20 afternoon sessions of 3-4 hours each. The themes were largely determined by the "multipliers" themselves, viz.:

3 afternoons:	Sets, subsets, equal sets, set bonds, teaching aids
2 afternoons:	Theory of sets, logic
1 afternoon:	Relations
1 afternoon:	Concept of number
2 afternoons:	Number bonds, addition to division
2 afternoons:	Concept of number (teaching methods)
1 afternoon:	Positional systems
1 afternoon:	Propositional functions

2 afternoons: Relations, especially order and equivalences, including teaching methods

1 to 2 afternoons: Teaching aids

1 to 2 afternoons: Basic concept of geometry and topology

1 to 2 afternoons: Text-books and work-sheets for the 1st school-year

1 afternoon: Preparation of subject; timetables.

Additional help was provided by the Baden-Württemberg Ministry of Education in the form of papers entitled "Mathematics in Primary Schools" (31) which were available to all primary-school teachers free of cost: their subject-matter was arranged, not in the order in which it would be presented in the classroom, but in its logical sequence:

Chap. I: Sets and propositional calculus

Chap. II: Relations, diagrams

Chap. III: Concatenations of graphs
Structural combinations (group structures)

Chap. IV: Natural numbers (combinations: +, .)
Positional systems, combinatory processes

Chap. V: Whole numbers

Chap. VI: Geometry.

These papers were certainly a great help: the manner in which they treated the problems involved was such as to stimulate motivation and reveal the elementary nature of the processes discussed, and for these purposes they were more suitable than any other literature then available.

Other measures

The training of "multipliers", coupled with compulsory in-service training at the afternoon sessions, each with its emphasis on particular points, was bound to clarify the new content of the mathematics syllabus. On the other hand, the way in which these courses were planned meant that "technical and scientific" advice on formal aspects and definitions was given too much weight. To combat the surfeit of theory, teachers taking a class in new mathematics for the first time were invited to attend a working party dealing exclusively with practical teaching problems, under the leadership of the same "multiplier".

In addition, the *Institut für Film und Bild* in Munich, in response to a suggestion from Baden-Württemberg, has recorded in pictorial form typical classroom situations arising from modern mathematics teaching in primary schools. The films are available and may be used at working party meetings.

Bavaria

In-service training for primary-school mathematics teachers takes place at three levels.

The Bavarian State Ministry of Education conducts a few central mathematics courses at the Teachers' Further-Training Academy in Dillingen (32) and other towns. They take the form of "summing-up" sessions (*Klausurtagungen*) and last a week. Those at the Dillingen Academy are partly aimed at the training of lecturers for courses held on a regional basis.

Secondly, the individual administrative areas (*Regierungsbezirke*) organise further-training courses on the subject of "New Mathematics in the Primary School" in accordance with local conditions and requirements. The distant-study course "New Mathematics for Primary School Teachers" (12) is also being started at regional level.

Finally, in-service training at local level is mostly carried out in the form of working groups.

Hessen

The Hessian *Institut für Lehrerfortbildung* (33) organizes further-training courses for teachers mainly in the form of one-week sessions. Three of these were held in the 2nd half of 1974, the theme for the 70 participating teachers being the outline directives on mathematics in the 3rd and 4th school-years.

At regional level, regular afternoon courses — 16 in all — were held in 1974 on the theme "Modern Mathematics in Primary Schools"; in the same year a series of working parties were also formed as an accompaniment to teaching activities.

Lower Saxony

Term-time courses

Central further-training courses are held in term-time. They are intended for primary-school teachers with no previous knowledge, and have been going on since 1971 in all departments of the Lower Saxony Teachers Training College and/or the Universities on the instructions of the State Ministry of Education. The number of participants per course varies between 50 and 150.

Programme

Participants are allowed one day off per week during term to attend a study session at the Teachers Training College or University. The proceedings begin with a 3-hour lecture in the morning, given by the mathematics lecturer; the afternoon is devoted to an exercise, in groups of about 25, led by members of secondary-school staffs or by trained and qualified primary teachers. In the following term there are no lectures, but the knowledge acquired is tested out in the classroom. Further lectures are given in the next term: the pattern of such a course thus consists of two one-term sections separated by a lecture-free term. The whole course totals 100 hours: 60 hrs of lectures (30 per term) and 40 hrs of exercises (20 per term).

Subjects covered

Technical information is given on set concepts, mathematical relations, sequences, arithmetic, positional systems, graphic topology, geometry, operators and groups; there is detailed treatment of the didactic reasoning behind the new syllabuses and the relevant battery of teaching methods, as well as questions of classroom technique and differentiation and comparisons between text-books; finally — and this is of great importance — the teachers attending exchange their own experiences thoroughly.

Courses for "multipliers"

In 1972 and 73 a selection of particularly well qualified teachers from each educational district attended a resident course at an institute of further training, in 3 stages of 2 weeks, 2 weeks and 1 week. There were between 30 and 40 participants at each course, whose contents were the same as those of the term-time courses, the participants being given additional training to enable them to run regional courses themselves.

Midway courses

Afternoon "midway" courses are organized at regional level by "multipliers" and other competent colleagues, such as seminar leaders and subject heads. They run throughout the school-year, being closely linked with classroom practice and sometimes with the actual text-book used. Visits are made to schools, practical experiences exchanged and common plans laid for teaching particular items from the syllabus.

In the participants' opinion many courses furnished too much theory without sufficient explanation in each case of the practical teaching problems. Teachers already possessing experience of new mathematics were especially insistent on the need for closer reference to practice.

Rhineland-Palatinate

There is a State Institute for the further and in-service training of teachers (34).

In the case of primary-school mathematics teachers the *Land* authorities rely almost entirely on the distant-study course "Mathematics for Primary-School Teachers" (12).

General planning

The Institute has held regional courses since 1972. Next year some 7,000 primary teachers will pursue their in-service training with the material provided by the distant-study course (12) under the direction of suitably trained "multipliers". The course is designed to cover 4 years: it is divided into a preparatory section, during which participants are introduced into the technical and material problems of new mathematics, and a more practical section designed to help the teachers concerned overcome the day-to-day difficulties of the classroom. The syllabus for the distant-study course (12) defines its aims.

Special planning: execution

1972

There were about 1,800 participants in all at the 62 regional courses held in 1972 (between 15 and 48 per course). In the 2nd half of 1974 the course was directed at 3rd-year mathematics teachers.

In 1972 each course included 10 full-scale meetings (*Tagungen*); in 1973 there were 8 and in 1974 five: four such meetings are planned for 1975 and 2 for the first half of 1976. Thus each annual course comprises 29 such meetings — 15 for grade 1, 6 for grade 2, 4 for grade 3 and 4 for grade 4.

1973

The 1973 arrangements also included 62 regional courses with about 1,600 participants: at each individual course the numbers ranged from 17 to 52. In the 2nd half of 1974 the target was mathematics teachers from grade 2.

The courses in this cycle included 7 full-scale meetings (*Direkttagungen*) in 1973 and 6 in 1974: 4 each are planned for 1975 and 1976, and 2 more for the first half of 1977. The total number of such meetings for each course in the cycle is thus 23 — 10 for grade 1, 5 for grade 2 and 4 each for grade 3 and grade 4 teachers.

1974

With the 1974 cycle 58 regional courses were started for a total of 1,400 participants who began grade 1 mathematics instruction in that year. They include the same number of full-scale meetings as the previous cycle, and will thus end in 1978 with two such meetings.

Regional courses of similar dimensions are planned to take place in 1975 for teachers who will be starting mathematics instruction in grade 1 in 1975/6; they will be completed in 1979.

The fact that the regional courses are closely linked with classroom instruction shows that the discussion of practical teaching problems is given pride of place. The material supplied by the distant-study course (12) assures the theoretical background.

In the meanwhile Rhineland-Palatinate has about 180 "multipliers" available, who carry out their task to the general satisfaction of participants at the regional courses. From these "multipliers", too, came the suggestion that the primary-school curriculum of 1971 was overloaded. The commission that has so far been responsible for preparing handouts on the mathematics syllabus for grade 1 consists predominantly of "multipliers".

Schleswig-Holstein

The in-service training of teachers is the responsibility of the *Landesinstitut Schleswig-Holstein für Praxis und Theorie der Schule* (35).

Schleswig-Holstein's effort may be gauged from the "Annual Report (*Jahresbericht*) 1972/73" (36). Its main findings are as follows: it has become constantly clearer that a satisfactory theoretical foundation is hardly possible, that the text-book used plays a central part, that technical and even secondary literature is not read and that the personality, i.e. the motivation of the course leader is of decisive importance, much more important in fact than the possibilities of central direction (37).

"Multipliers"

In the 1972/73 school-year 12 teachers were partly freed from their normal duties to act as in-service training leaders. They met together for 1 or 2 weeks for their own training, exchanges of experience and joint work, including the production of material and the analysis of text-books. The teachers thus freed, for their part, offered at least two courses, taking place at fortnightly intervals in the afternoons. Other course leaders, who had gained practical experience in recent years, were requested to continue their training activities, and took part in some of the joint working sessions, so that to date it has been possible to dispense a sound training to some 20 leaders in the course of weekly or fortnightly meetings of one kind or another. At present the leaders meet every 4 weeks to co-ordinate the work. Some of them are full-time subject heads at area seminars; almost all of them are called upon by the local school boards as advisers and examiners, and they take charge of regional courses.

Regional courses

About 25 courses linked with classroom instruction and current text-books and entitled "Mathematics in the primary school" are at present being held as an aid to teachers of grades 1 and 2. As a general rule one afternoon sitting takes place every two or three weeks.

Crash courses

In addition, 1974 saw the introduction of eight one-week crash courses designed to prepare teachers who would be starting mathematics in grade 1 as from Easter. About 350 teachers in all attended these courses, the prime movers in which were the Teachers Training Colleges (38). It is intended that in 1975 similar courses shall be held for teachers of grades 2 and 3.

Regional shorter courses

A new formula for regional in-service training is the following: To begin with, the syllabus for grade 1 was worked out in co-operation with the directors of studies. Three rapporteurs were found for the 6 areas of study covered. A course was then held in the form of one morning and 4 afternoon meetings, the 6 areas being dealt with in 4 weeks in close liaison with the classroom. At the beginning of the school-year about 300 teachers took advantage of this offer, which constituted a practice-oriented survey of mathematics teaching in grade 1. The initiative thus taken by the training leaders found a wide echo.

It is worth noting that there is a clearly-marked trend towards regionalization of in-service training. Experience shows that it is not possible to train multipliers for a single school, but in-service training arrangements are nevertheless planned for 2 or 3 larger schools. The idea is to exchange visits and prepare teaching programmes and proposals which are likely to have a major influence on classroom practice.

Conclusions: Summing-up

Problems of training

The problems discussed in the foregoing apply to all subjects, but their solution can be followed up in the case of mathematics, where they have become particularly urgent today. As regards the training of primary-school teachers it is here found that about 10 % — those who took mathematics as a main subject — have adequate technical knowledge, and that 30 %, i. e. those who took mathematics as a special or a subsidiary subject, have as a rule adequate training, though this does not apply to all cases. Practical teacher training in the subject "mathematics" at the *Bezirksseminar* is thus received only by the above-mentioned 10 % and about half the other 30 %, i. e. altogether some 25 %.

Even if the percentage is higher — it is certainly roughly of that order — it is clear that about 70 % of primary-school teachers are not adequately prepared for mathematics instruction. No account is taken here of the problem of teachers trained in accordance with the old canon.

It is hardly conceivable that this deficiency can be made good by further-training methods within the framework of existing institutions, however wide their scope. Even if we restrict ourselves to assessing present capacity, the further training of teachers can, by definition, be no more than a "further" measure, i. e. a measure designed to "top up" an assumed level of training to the current state of the art, be it in the technical, pedagogical, psychological, didactic or methodological fields – the pre-requisite being corresponding teacher-motivation; in the final analysis, no further-training arrangements can do more than help the teacher to work out his own salvation – they cannot make up for any initial training that may have been missed.

Perhaps the solution to this problem may lie in some degree of teacher specialization as early as the 1st grade of primary school, or in certain restrictions on the free choice of subject by future teachers at the stage of their studies.

The relations between the phases through which every teacher passes in succession, viz.:

Phase 1: Studies

Phase 2: Preparatory service

Phase 3: Actual teaching, followed by further or in-service training

are so far somewhat theoretical: there is recognition of their necessity but little more. We have not yet reached the stage of a proper feedback between various parts of the individual phases, especially as there are strict limits to the number and scope of the possibilities open in phase 3.

For some time past the subjects offered by the various higher education establishments have included generous provision for "New Mathematics", and the practical training undergone by student teachers during their preparatory service should by now be regarded as satisfactory.

General problems of in-service training

Voluntary participation; "time off"

Voluntary participation means that the teachers concerned have a definite motivation, but there is also the risk that these are precisely the "wrong" teachers — the courses will consist in preaching to the converted. Where reforms are in the air this is undoubtedly a problem, but the principle that in-service training is freely offered and voluntarily accepted continues to be the rule in the Federal *Länder*.

In that connection the question of "time off" — i. e. the transfer of part of the teacher's service obligations to courses of further training — is becoming acute, since the successful achievement of reform necessarily calls for an impersonal interest by the *Land* authorities in an in-service training system.

Up to now, no such "time off" has been granted, except for the two-year courses in Bremen.

Problem of "multipliers"

Even if only "real further training" in the sense just mentioned is intended, the present capacity of the institutions hardly suffices. In any event, they cannot cope adequately with *direct* training. Hence the need for graded training using some form of "multiplier" system. The weak point of all such systems lies, as has been shown, in the multiplier himself, for — inter alia — the number of suitably qualified teachers is certainly limited, and as the need for "multipliers" grows so do the dimensions of this problem.

Special problems of in-service training in mathematics

It has been broadly characteristic of all further-training schemes that they begin by offering subject-information as a foundation for possible transposition to classroom practice. Here, obviously, it is curriculum content – the material aspect – which holds the forefront. This points to special dangers in the case of mathematics, in that gaps in knowledge can be precisely gauged and followed up. But subject-matter is only one component of reform – a necessary component, admittedly, but unfortunately it is not enough in itself, nor is it the most important element. The disadvantages of proceeding in this manner may be summarized as follows:

1. The difficulties encountered by many teachers in "do-it-yourself" learning are much greater than expected. (The concept of "set" and "element", the relations "E" and "C" and the introduction of the "average set" completely fill a whole day of lectures!)
2. Advance ex cathedra technical information is not accepted by many teachers, who find that it takes up too much time and is of dubious value professionally in the light of its subsequent usefulness and applicability. Many feel that they are overloaded and consider "New Mathematics" to be a difficult and abstract thing; if they are compelled to teach it, their pupils experience the same feeling to an even greater extent. The wave of criticism observed since 1974 has caused uncertainty in many sections of the teaching body, so that readiness to receive information has visibly decreased, although one cannot avoid the impression that the criticism is sometimes also used as a convenient alibi.
3. There was, in addition, the risk of a misunderstanding in the shape of a widespread feeling that the nature of the reform lies in the content of the mathematics to be taught ("content" being here taken to include mathematical methods): in other words, new content was simply exchanged for old, but the conventional type of teaching remained. Sometimes, too, technical information – through a simple displacement of level – was transferred direct, i. e. in the same order, to the classroom.
4. The opposite effect was also found. Some teachers, who by dint of great efforts and maximum receptivity fully assimilated the technical information offered, gave evidence of a kind of euphoria;

this was no doubt due to satisfaction at their achievement, at the cultural experience which it represented, and also to a touch of the pioneer spirit. The great danger here lies in overestimating the importance to teaching of the new mathematical concepts and methods, which may be blown up to the dimensions of a panacea.

5. The over-valuation of theory weighed especially heavy in the balance because it transpired that teachers in general find it very hard to accomplish a methodical transposition of technical knowledge in a new field. On this point the guidelines were found to be of small help, if indeed they were known at all.

6. Accordingly, alongside the provision of technical information an attempt was made at direct analysis of possible methods – backed up, where feasible, with demonstration lessons. This closer reference to practice was found by the teachers to be necessary, meaningful and beneficial – which, in a word, was tantamount to saying that it saved them a certain amount of "teaching preparation". Closer reference to practice is, however, a contribution to the desired reform only in so far as it can change the attitude of the teacher.

7. It was found that the vast majority of teachers have no access to literature on teaching methods and seek none. Their reading in this direction is often restricted to the teacher's manual that accompanies a mathematics text-book – and this in its turn increases still further the already considerable influence of the school text-book. The realities of school life are, in fact, much more strongly determined by the text-book market than by guidelines and education committees.

"School text-book production is guided, for its part, by directives or outline directives which in their turn are drafted by groups generally appointed by the education ministries. What actually goes on in the classroom is, however, sometimes so widely different from these rules that some thought must be given to the question of how this side of our school system can be improved. Be that as it may, the influence on school realities of the present guidelines, so far as they go beyond the mere delimitation of subject-matter, is, in my opinion, a very modest one" (39). In future the in-service training of teachers must include an effort to secure closer co-operation between teachers, schools, administration and research workers, all these groups mutually influencing the others.

Here, indeed, lies the heart of the problem. Broadly, as we have pointed out elsewhere, the teachers have favoured a reform of content. They have hardly ever, or all too seldom, seen it as a question of teaching reform, a reform of ways and means of instruction. The reason may be that the reform envisaged was too closely on the lines of the "Planning-Research-Development-Diffusion" model. Corresponding models have failed, for example, in the USA. "To suggest a perfect form of teaching material to the teacher, i. e. to use him exclusively in the way the builders of the system require, has proved to be a sheer impossibility" (40). But the curricula, or their reflection in the teaching manuals available, can certainly not be regarded as perfect teaching material. A teacher's mathematical knowledge can be influenced without overmuch difficulty, but his behaviour while teaching, his customary ways of taking a class, his ideas of interaction-patterns between teachers and pupils, i. e. his whole self-awareness, are far less amenable to outside pressures.

And that, apart from securing certain marginal results of a partly organizational nature, such as the size of classes, is what in-service training is all about.

How can we achieve success?

There is, alas, no royal road. Alongside the conditions already mentioned one can only say that "further training" must in no case begin only at the "diffusion" stage, in accordance with the model referred to above: reform must be carried out concordantly at all the various levels – pupils, teachers, researchers, classroom, school boards, curriculum, text-books – and properly timed in consequence, otherwise it will fail. The whole aim of in-service training, by exploiting the best possible means available at any given time, must be to contribute towards making it a success.

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(3) In (2): p. 42.

(4) In (1): p. 485/486.

(5) Krautmacher, W.: KMK — Empfehlungen im Spiegel der Länderrichtlinien (Versuch einer Interpretation) (The KMK "Recommendations" in the light of the *Länder* guidelines – an attempted interpretation). Die Grundschule. 4. (1972) 132–140.

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(7) In (6): p. M/2.

(8) In (6): p. M/10.

(9) Cf. (7), first part. Page 110.

(10) Griesel, H. and Hollmann, E.: Eine mathematisch-didaktische Analyse dreier Erstrechenkurse (A mathematic/didactic analysis of three maths. courses for beginners). Zentralblatt für Didaktik der Mathematik 1 (1969) 32—39.

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(12) Mathematik-Kurs für Grundschullehrer
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(13) In (12): p. 5.

(14) In (12): p. 5/6.

(15) In (12): p. 8.

(16) In (12): p. 8.

(17) In setting up the courses for directors of studies in the districts administered by the *Regierungspräsidenten* in Düsseldorf and Cologne, the *Institut für Lehrerfortbildung* in Essen-Werden, an establishment run by the Catholic Bishoprics in North Rhine-Westphalia, played a decisive rôle.

(18) Cf. E 1: "Sets and their presentation" – Technical and didactic sections: p. 5, bottom.

(19) Bauersfeld, H.: Wege zu einer dezentralisierten Curriculumentwicklung (Towards decentralized curriculum development). In: Regionale Pädagogische Zentren in der Diskussion. Essen, 1973, Stifterverband für die Deutsche Wissenschaft e. V. p. 20.

(20) Cf. passage on pp. 8/9 of the present paper, quoted from the "Richtlinien und Lehrpläne für die Grundschule in Nordrhein-Westfalen" (6).

(21) Cf. Resolution adopted on 21/22 May 1974 on "Primary-School Reform and Mathematics" by the *"Arbeitskreis Grundschule e. V."*
In: Die Grundschule: Grundschulreform und Mathematik 1974, pp. 3–5.

(22) Bill tabled by the Berlin Senator for Education, No. 2922/74 of 2 July 1974: "Bericht über den Unterricht in der Mengenlehre ("Neue Mathematik") an der Berliner Schule" (Report on the teaching of the theory of sets ("New Mathematics") in the Berlin schools).

(23) On this point see pp. 45–46.

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(This is not the Freiburg distant-study course mentioned in n. (12), which at that time was not yet ready, nor was its draft viewed with much enthusiasm by the responsible parties in Baden-Württemberg.)

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(35) Landesinstitut Schleswig-Holstein für Praxis und Theorie der Schule
D-2300 Kronshagen bei Kiel, Schreberweg 5.

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(39) In (19): p. 21.

(40) In (19): p. 20.

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